ACOUSTIC WAVE SCATTERING FROM DYNAMIC ROUGH SEA-
SURFACES USING THE FINITE-DIFFERENCE TIME-DOMAIN
METHOD AND PIERSON-MOSKOWITZ FREQUENCY SPECTRUM

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Abstract: Most models for underwater acoustic propagation typically assume the sea-surface
to be either perfectly smooth or rough but “frozen” in time. Long duration transmissions on
the order of tens of seconds are being considered for next-generation sonar. These types of
signals improve target resolution and tracking. However, they can interact with the sea-surface
at many different wave displacements during transmission. This violates the frozen surface
assumption and causes anomalies in the received signal which introduce additional
transmission losses and Doppler effects. Full wave propagation models can be used to better
understand the mechanisms behind these anomalies. This understanding leads to better system
design and enhanced performance without having to perform expensive at-sea experiments. In
this paper, a finite-difference time-domain (FDTD) method is developed to model the impact of
both roughness and motion of the sea-surface. The FDTD method is a full-wave numeric
 technique that allows an arbitrary function to define the boundary points within the
computational space. Surface motion is accomplished by modifying these boundary points at
each time step. The rough, time-evolving sea-surface is modeled using a Pierson-Moskowitz
(PM) frequency spectrum, which is simple to implement and fully defined by wind speed and
direction. Results from FDTD simulations of static rough sea-surfaces are compared to a
previously established integral equation solution method to evaluate the validity of the
approach. Agreement is also demonstrated for FDTD simulations of a dynamic rough sea-
surface and a theoretic statistical model. [Work supported by the Office of Naval Research]

Keywords: Underwater acoustic propagation; Finite-difference time-domain; Pierson-
Moskowitz frequency spectrum; Dynamic rough sea-surface
INTRODUCTION

Over the past few decades interest has increased in creating an internet of underwater things[1] (IoUT). Doing so would greatly alleviate difficulties in underwater exploration and monitoring by removing the necessity of tethering underwater vehicles and sensor systems. Due to rapid attenuation underwater electromagnetic solutions cannot be directly transferred from land to underwater channels. In this regime acoustic modem are predominately used for communications. These types of systems are hindered by their high bit error rates and long propagation delays. Additionally, sound speed variations in depth can cause shadow zones and loss in connectivity. Due to the large coverage areas and variations in the channel, measurements are not feasible. Modeling techniques are therefore needed to offer insights and drive the design of acoustic communication systems that can mitigate these issues.

Long duration signals on the order of tens of seconds are being considered for the next-generation[2] of SONAR. These types of signals improve target resolution and tracking. However, they interact with the sea-surface at many different displacements along the sea-surface during their transmission. This violates the frozen boundary assumption of most modeling techniques and causes anomalies in the received signal that add to transmission losses and introduce Doppler effects. Adding realistic motion to the sea-surface boundary would increase the accuracy of the model and provide insights into the mechanisms of the observed anomalies. These insights drive decisions that help SONAR designers build more robust systems.

A modeling technique that is capable of handling boundary roughness and motion is required. The finite-difference time-domain (FDTD) method fits these requirements. In the next section, the FDTD method for scalar acoustic pressure fields is discussed. In this section model results from a modified Image method (M-IM), Integral equation (IE) approach and Kirchhoff approximation (KA) are compared to FDTD simulations. Then, random rough sea-surfaces are discussed. Models capable of simulating static random roughness are compared. Then the spectral approach[3] proposed by Pierson-Moskowitz (P-M) is combined with the FDTD method to simulate realistic dynamic rough sea-surfaces.

FINITE-DIFFERENCE TIME-DOMAIN METHOD

The finite-difference time-domain method is a well establish numeric technique that is used to approximate the wave equation. In its general form it can be applied to solve any ordinary and partial differential equation. It has been used extensively in electromagnetics to simulate Maxwell’s equations[4]. Typically used for propagation and scattering problems that include complicated boundary conditions or geometries, it offers insights as to how fields interact spatially and temporally and is capable of ultra-broadband solutions in a single simulation. The FDTD method has also been used in seismic exploration and in underwater acoustics. Practical examples include; seismic propagation around faults zones or areas which contain potential oil and gas traps or acoustic interactions with elastic bottoms and scattering from geometric irregularities such as keels and leads on the underside of Artic ice cover.

In this section the wave equation which governs seismo-acoustic propagation is implemented using central finite-differences over a two-dimensional mesh that represents an ocean half-space, as shown in Fig. 1. The two dimensional (2D) scalar wave equation for an acoustic pressure field produced by a transient line source is

\[
\frac{\partial^2}{\partial x^2} p(x, z; t) + \frac{\partial^2}{\partial z^2} p(x, z; t) - \frac{1}{v_c} \frac{\partial^2}{\partial t^2} p(x, z; t) = -S(t)\delta(x-x_s)\delta(z-z_s),
\]  

(1)

where \( p \) is the acoustic pressure, \( x \) and \( z \) are spatial coordinates within the computation grid, \( t \) is time, \( v_c \) is the acoustic velocity, \( S(t) \) is an arbitrary time dependent line source function and
\( (x_S, z_S) \) are the source position. Equation (1) is a second-order partial differential equation describing acoustic propagation. By discretization of the independent variables the differential operators are locally approximating by finite-differences. Discretization for the ocean half-space is shown in Fig. 1-b. Where a rectangular mesh over the entire computation domain is defined as \( (x_i, z_j; t_n) = (ih, jk; n\Delta t), i = 0 \cdots N, j = 0 \cdots M \) and \( n = 0 \cdots T \).

\[ p^n_{i,j} = u^2(p^n_{i-1,j} + p^n_{i+1,j} + p^n_{i,j-1} + p^n_{i,j+1}) + 2(1 - 2u^2)p^n_{i,j} - p^{n-1}_{i,j}, \]

where \( n \) is the time-step index, \( i \) is the range index, \( j \) is the depth index and \( u = \nu c n / i \). The right-side of (2) is referred to as the updated part of the equation. The acoustic pressure at the next time-step \( t_{n+1} \) gets updated with the finite-difference of the current time-step \( t_n \) and the previous \( t_{n-1} \). In this manner, the field is approximated for all grid points in the mesh and marched/stepped forward in time. At any given time-step only knowledge of the current and previous time-step are needed, with exception to the boundaries along the left \((x = x_0, z)\), bottom \((x, z = z_M)\) and right \((x = x_M, z)\) edges of the mesh. These are special cases for (2) in which there is no adjacent pressure value.

The ocean half-space model (Fig. 1) dictates that the boundary along the sea-surface be pressure-release. This is implemented using a Dirichlet boundary condition, effectively maintaining that the mesh points along the sea-surface are zero. This will ensure that (2) is unbalanced at \( p^n_{i,j+1} = 0 \). Resulting in the next time-step being offset by the same amount. This is analogous to reflection. Along the left, right and bottom edges we have arbitrary boundary conditions. For all three of these cases the field needs to continue propagating as if it did not encounter a boundary. To enforce this type of behaviour while maintaining the integrity of the incident pressure point a perfect-matching layer (PML) is implemented around the edges. This type of boundary condition requires a minimum of two additional prior time-steps in order to cancel only the reflected amount at the boundary point. This is analogous to rapidly attenuating the field over two time-steps.

A high-level algorithm for the 2D FDTD simulations presented throughout this work is summarized in Table. 1. A baseline 2D FDTD simulation of a smooth static sea-surface is
<table>
<thead>
<tr>
<th>Step</th>
<th>Description</th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>Determine highest acoustic frequency of interest</td>
</tr>
<tr>
<td>2</td>
<td>Initialize the computation domain</td>
</tr>
<tr>
<td>3</td>
<td>Determine arbitrary source function</td>
</tr>
<tr>
<td>4</td>
<td>Determine finite-difference for each mesh point at time-step $t_n$</td>
</tr>
<tr>
<td>5</td>
<td>Store points of interest for post-processing</td>
</tr>
<tr>
<td>6</td>
<td>Advance pressure fields, i.e. $p_{i,j}^{n+1} = p_{i,j}^n$</td>
</tr>
<tr>
<td>7</td>
<td>Inject source function value at time-step $t_{n+1}$</td>
</tr>
<tr>
<td>8</td>
<td>Repeat steps 3 – 6 until event of fields has occurred.</td>
</tr>
</tbody>
</table>

Table 1: High-level finite-difference time-domain algorithm.

shown in Fig. 2. The source function is a unity amplitude sinusoidal tone $f_s = 375$ Hz located at $(x_s, z_s) = (-90, -32)$. A Tukey window was used with $\alpha = 0.075$ to ease the beginning and ending transitions of the source function. Using a wavelength divisor of 10 sets the mesh spacing equal to 0.4 m. A rectangular mesh was chosen for simplicity, other approaches are available but are outside the scope of the work present here. To enforce stability of the PML boundary conditions the acoustic wave needs to propagate one wavelength over two time-steps $\Delta t = 0.4/(2\nu_c) = 0.133$ ms. After $t = 21$ ms the source has radially propagated 32 m and exhibits 30 dB transmission loss due to geometric spreading. This is observed in Fig. 2a where the field $\sim 12$ m away from the source is in the $30 - 35$ dB range. At $t = 0.168$ ms the field along $z_s$ has reflected at the sea-surface and propagated 252 m. This is seen the Fig. 2b where the field extends beyond the computation space at $z = -200$ m. The total pressure clearly shows a pattern of nulls due to interference by the direct path from the source and the sea-surface multipaths. Following a sea-surface reflected path from the lower bottom-right corner of the computation space to the source traces out $\sim 232.35$ m. This is less than the distance previously stated directly under the source. Fig. 2b demonstrates this behaviour as the extent of the wave front is in the bottom-right corner of the computation space. When the simulation reaches a steady-state the entire computation space is filled with the total field and the familiar Lloyds Mirror pattern is easily identifiable in Fig. 2c.
This same scenario happens when the simulation is run and the frequency is doubled (Fig. 2d).

The 2D FDTD implementation is further verified by comparing the results to three other numeric techniques; Image method[6], Kirchhoff approximation and Integral equation[7]. These techniques numerically solve the Helmholtz equation with solutions in the time-harmonic form. Finite-difference time-domain simulations must reach steady-state before the results can be compared to these models. This is demonstrated in Fig. 3 for the same tonal source function located at \((x_s, z_s) = (0, -32)\). All three techniques exhibit the expected interference pattern by a static smooth sea-surface. The IE and KA have less than 0.04 % average difference across the computation space. The Image method has slightly more overall transmission loss which can be attributed to normalization errors introduced by a singularity at the source point. Additionally, the FDTD method shows less transmission losses directly below the source due to the continuous time source function.

With confidence that the 2D FDTD implementation properly simulates the acoustic physics of the ocean half-space a more realistic sea-surface model is desired. In the following section two random rough sea-surface models are discussed. First static sea-surfaces generated by a Gaussian roughness model then a fully developed wind sea model produced by P-M frequency spectrum.
Fig. 3: Comparison of pressure field determined by different models for a static smooth sea-surface \( t = 0.082 \) s, \( f_s = 375 \) Hz.

**RANDOM ROUGH SEA-SURFACES**

A more realistic sea-surface model would include roughness and motion to mimic gravity waves produced by the fluctuations of fluids. To start the sea-surface is extended to the static Gaussian roughness model[6] and then to a fully developed wind seas model as described by the P-M spectrum[3].

The Image method is unable to handle scattering due to roughness along the sea-surface boundary. This is implicit in its formalization which only takes into account specular reflections by placing the source “image” at a mirrored position above the boundary. The IE and KA include scattering terms and can be compared to 2D FDTD simulations that include a random Gaussian roughness model for a static sea-surface. Both the IE and KA provide solutions that are time-harmonic and therefore are currently unable to handle dynamic sea-surfaces.

The static Gaussian rough sea-surface is defined by a root-mean-square wave height \( h_{rms} = 0.212 \lambda \) and coorelation length of \( l_x = 0.9 \lambda \). The techniques compared use the same static Gaussian rough sea-surface. The computation grid has been limited to \( x = [-70, 70] \), \( z = [-140, 0] \). The same source function and position were used as in the prior section. The steady-state field of the FDTD simulation occurs at times \( t > 0.124 \) s.

(a) Integral equation method \( t = 0.167 \) s
(b) Kirchhoff approximation \( t = 0.167 \) s
Fig. 4: Comparison of pressure field determined by different models for a static rough Gaussian sea-surface $f_s = 375$ Hz.

Discrepancies in the FDTD field and the IE and KA in Fig. 4 are due to slight differences in $\Delta t$. In the FDTD case $\Delta t = 0.4/(2\nu_c) = 1.333$ ms and for the IE and KA $\Delta t = 1.3$ ms. This causes a phase offset of 3.33 $\mu$s between the fields in Fig. 4a-b and Fig. 4c which is less than one FDTD time-step.

The P-M spectrum approach is well suited for modeling rough, time-evolving sea-surfaces. It is relatively simple to implement and fully defines the sea-surface by wind speed and direction. Preliminary results for 2D FDTD simulations using a P-M spectrum to generate a dynamic rough sea-surface boundary are shown in Fig. 5. The same tonal source function located at $(x_s, z_s) = (-90, -32)$ was used for these simulations. The P-M spectrum was generated using a wind velocity $v_w = 22.6$ m/s and a dominant wind direction $\phi_w = 0^\circ$. These parameters were chosen to mimic a Beaufort Sea State (SS10) which is considered storm condition wind forces. Using an extreme case accentuates the motion of the sea-surface over the relatively short duration of simulation. For the simulations in Fig. 5 the peak wave travels approximately 12 m in the positive x-direction. This is further indicated by the static white x-marker located at $(x, z) = (90, -128)$ in Fig. 5b-c which passes through a null as the sea-surface minimum advances.
CONCLUSIONS

The finite-difference time-domain method has been implemented for acoustic wave propagation and scattering within an ocean half-space. This technique has been compared to other numeric methods for smooth and rough Gaussian static sea-surfaces. The IM is unable to handle specular scattering due to its requirement of placing a fictitious source at a mirrored location above the sea-surface boundary. The IE and KA are well suited for static random sea-surfaces but lack realistic surface wave motion in their formalizations and suffer from potential loss in generality due to conversions from their time-harmonic solutions. The FDTD method determines the pressure field at a marched-step time indices which easily accommodates boundary motion. The well-established P-M frequency spectrum provides realistic dynamic sea-surfaces and has been used incorporated into FDTD simulations. Insights gained by FDTD simulations with realistic sea-surfaces provide a practical and cost effective way to guide design decisions for acoustic modem and next-generation SONAR systems.

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REFERENCES