RANGE DEPENDENT TRANSVERSAL FLOW RETRIEVAL BY A MULTI-FREQUENCY ACOUSTICAL APPROACH

Igor Esipov

Russian oil and gas university/Andreyev Acoustics institute, Leninskyprosp., 65, Moscow 119991, Russia
Fax: +7 (499) 507-88-77, e-mail: igor.esipov@mail.ru

Abstract: Transverse flow of inhomogeneous current produces fluctuation of the acoustic signal passing through it. These fluctuations vary with the CW signal frequency change due to variation of the Fresnel zone size. Respectively, the fluctuations of amplitude and phase of frequency-spaced acoustical signals are coherent at the low frequency and no coherent at the high-frequency band of fluctuations. The frequency cutoff of the coherence function depends upon the flow velocity at given fine structure of the turbulent flow. The measurement of the cutoff frequency allows determining the transverse current velocity. This method can be considered as a frequency-domain version of the conventional scintillation approach to the current velocity. It efficiency was verified earlier at the example of tidal current retrieval for the field research at Cordova Channel (Canada). Now we present here results of the experimental research of multi-frequency acoustical technique implementation to retrieve both the flow velocity and the spatial position of the transversal turbulent stream in a quiet environment. Besides the results of experimental research the paper presents a brief theory review of the method both and computer simulation of multi-frequency signal propagation through the variable inhomogeneous medium. Hereby prospective multi-frequency acoustical technology for ocean observatory is discussed.

Keywords: Multi-frequency sound propagation, Signal fluctuations, Multi-frequency signal coherence.
INTRADUCTION

Sound signal, passing through the turbulent flow, obtains fluctuations both amplitude and phase. Such fluctuations called as scintillations as well. Experimental research [1] and computer simulation [2] shows the efficiency of inversion procedure based on sound scintillation approach. Correlation technique for the signals propagated along spatially separated paths is applied in this case. When spacing between sound passes doesn’t exceed frozen turbulence scale, fluctuation time delay directly related to transversal flow velocity. Well-developed acoustical tomography method needs multiple spatially separated paths and spatial resolution for the environmental inversion is directly related to number of such paths. The complex and variable environment in ocean and especially in Arctic Ocean makes it very expensive and unreliable to apply conventional acoustical tomography technique. This paper is proposed to develop one-path tomography scheme to monitor the transverse current flow based on investigation of signal scintillation in frequency domain [3]. Therefore the proposed multi-frequency approach promises to provide the tomography inversion in the complex ocean environment with reduced number of acoustical paths. Earlier multi-frequency approach was experimentally tested for tidal current retrieval in the field research at Cordova Channel (Canada) [4]. Hereby we present the results of experimental research of range dependent transversal flow retrieval by a multi-frequency acoustical approach.

THEORETICAL BACKGROUND

The transversal turbulent flow produces fluctuation of the acoustic signal passing through it. These fluctuations vary with the CW signal frequency change due to variation of the Fresnel zone size. Respectively, the fluctuations of amplitude and phase of frequency-spaced signals are coherent at the low frequency of fluctuations and no coherent at the high-frequency band. The frequency cut off of the coherence function depends upon the flow velocity at given fine structure of the flow. The measurement of the cut off frequency allows determining the transverse current velocity [3,4]. Let us consider the propagation through the random medium of the n CW waves of different frequencies: \( \omega_n \) and \( \omega_m \). Normalised cross-spectrum (or coherence) for log amplitude \( \chi \) or phase \( S \) of the signals with carried frequencies \( \omega_n \) and \( \omega_m \) is:

\[
\Gamma^{(n,m)}_{\chi,S} = \frac{W^{(n,m)}_{\chi,S}(\nu)}{[W^{(n,n)}_{\chi,S}(\nu)W^{(m,m)}_{\chi,S}(\nu)]^{1/2}}
\]

Here \( W^{(n,m)}_{\chi,S}(\nu) \) - cross-spectra of amplitude or phase fluctuations between the signals of different frequencies, and \( W^{(n,n)}_{\chi,S}(\nu) \) and \( W^{(m,m)}_{\chi,S}(\nu) \) corresponding power spectra, \( \nu \) - signal fluctuation frequency.

Simple analysis for turbulent flow restricted in some compact area, shows that cut off frequency \( \nu_c \) for the coherence function (1) will be dependent on the spatial position \( x_0 \) of turbulent streem and the value of it velocity \( U \).
\[ v_c \approx U \left( \frac{kL}{x_0(L-x_0)} \right)^{1/2} \tag{2} \]

where \( k = 2k_n k_m / (k_n + k_m) \). To solve arising ambiguity in definition of both \( U \) and \( x_0 \) one could use any additional independent information, for example time delay \( \Delta \tau \) between the signals of the same frequency, but received by the pair of spatially separated gages. That time delay will be the other function of \( U \) and \( x_0 \).

\[ \Delta \tau = \frac{bx_0}{UL} \tag{3} \]

here \( b \) is a spatial gap between the gages and \( L \) is the path length. Therefore we obtain a couple of independent relations (2) and (3), which define unambiguously the value \( U \) and the current position \( x_0 \). It is necessary to take into the mind that frozen turbulence model application is restricted for turbulence drift only on several turbulence spatial scales. This condition leads to estimation \( v_c \Delta \tau \approx 1 \) which means the reliable spatial gap between the gages couldn’t exceed \( b \approx (L/k)^{1/2} \), or gages should be spaced approximately at Fresnel zone dimension. For larger separation, signals received by the different gages can lose their coherence, for smaller separation the signals will be quite coherent and condition (3) loses it sense.

As a pair gages are separated in that method no more than one Fresnel zone dimension, formally we could say in this paper about one-spatial path acoustical scheme. Cross-section of each sound path should be evaluated namely by a Fresnel zone dimension and therefore we could consider them overlapped. We discuss here the principals and key features of multi-frequency one-spatial-path tomography approach to retrieve the transverse current flow based on investigation of signal scintillation in frequency domain. This principal of multi-frequency range dependent turbulent flow retrieval has been realized experimentally in laboratory condition.

**EXPERIMENT**

Experimental research has been done in air. Loudspeaker was used as a source of multi-frequency sound signal (Fig.1). Sound signal was received by a chain of 8 microphones spaced with a step of 5 cm symmetrical with respect to path axis and placed at distance \( L=7.75 \) m from the source. Turbulent flow across the sound path was generated by a fan installed at \( x_0 = L/2 \) or \( x_0 = L/4 \) from the source. The width of the flow was about 1m at the sound path. The flow velocity \( U \) was measured by a cup anemometer and changes from 1.22 m/sec to 1.81 m/sec. Loudspeaker simultaneously transmitted a continuous signal with four discrete frequency components at 8 kHz, 10 kHz, 12.5 kHz and 16 kHz. Turbulent flow produces both phase and amplitude modulation of the signal. Fig.2 shows the effect of turbulent flow on spectrum of one of the frequency component of the signal. Signal fluctuations at each frequency components were processed to determine the cross-correlation and coherence functions in accord to Eq. 1. Experimental condition provides a sufficient signal-to-noise ratio only for maximum velocity of turbulent flow. Records of the signals lasted at least of 1 min for statistical confidence.
For the further analysis of the signal coherence we introduce dimensionless frequency $FF = F(\Omega)^{1/2}$ where

$$\Omega = \frac{\omega_n - \omega_m}{\omega_n + \omega_m}$$

- dimensionless frequency carrier frequencies $\omega_n$ and $\omega_m$, $F = \nu / \nu_0$ is the ratio of current frequency of cross-spectrum to some constant frequency. As the frequency of sound modulation by the turbulence flow will be dependent on the ratio of flow velocity to Fresnel zone size, accord to Eq. 2, we choose the constant frequency as $\nu_0 = \frac{2}{\kappa} \frac{L}{x_0}$

Such choice of $\nu_0$ leads to dimensionless cut off frequency became the same for all signals $FF_c \approx 1$ (Fig. 3).

To retrieve characteristics of the turbulent flow one needs in signal of at least two carrier frequencies $\omega_n$ and $\omega_m$, determine dimensionless frequency $\Omega$. Further it is needed determine cut off frequency for coherence function for signal fluctuation at both frequencies $\nu_c$ and determine dimensionless value $FF_c$. In our case $FF_c = 1$. This result should be added by the result of time delay measurements $\Delta \tau$ between fluctuations of the signals of the same frequency, but registered by the different gages. Accord to Eq. (2,3), that data determine the system of two equations, which solution gives $U(x_0)$ and $x_0$.

$$\frac{\nu_c(\Omega)^{1/2}}{FF_c} = U(x_0) \left( \frac{kL}{x_0(L-x_0)} \right)^{1/2}, \quad \Delta \tau = \frac{b x_0}{U(x_0)L}$$

(4)
Fig. 3. Signal coherence for different pairs of frequencies with respect to dimensionless frequency $FF_c$; a) – experiment, b) – modeling for different fluctuation spectra $\delta\epsilon(x, y)$. 

Fig.3 shows that the cut off of the experimental coherence functions are very close each other at the dimensionless frequency $FF_c \approx 1$ for different pairs of current frequencies. This result corresponds to the conclusion from [3].

To check the experimental choice of $FF_c = 1$, the computer simulation of this process has been done. An approach referred in [5] was used for the simulation, where turbulent flow is regarded as a random sum of plane oscillations $\delta\epsilon(x, y)$ with Kolmogorov-Obukhov spatial spectrum confined in a restricted area and moving with a constant velocity. To verify the method feasibility to type of turbulence structure, three types of $\delta\epsilon(x, y)$ power spectra with power low of (-3/3), (-5/3), (-7/3) have been investigated. As it was shown at Fig. 3b), the cut off frequency is independent to type of turbulence. The results of experimental research and computer simulation show that our choice for the value of dimensionless cut off frequency $FF_c = 1$ is confirmed with sufficient accuracy. This value is stable when turbulence spectrum variation in very wide range for power dependence in turbulence spectrum from -1 to -7/3. Results of $U$ and $x_0$ inversion are shown at Table 1.

<table>
<thead>
<tr>
<th>$U$ (m/sec)</th>
<th>1.22</th>
<th>1.39</th>
<th>1.81</th>
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<td>$x_0$, m</td>
<td>3.87</td>
<td>1.94</td>
<td>3.87</td>
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Results of the inversion with respect to all pairs of frequencies

<table>
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<th>$U$ (m/sec)</th>
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<th>1.3</th>
<th>1.5</th>
<th>1.4</th>
<th>1.5</th>
<th>1.3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_0$, m</td>
<td>3.7</td>
<td>1.8</td>
<td>4.3</td>
<td>1.99</td>
<td>3.3</td>
<td>1.7</td>
</tr>
</tbody>
</table>

Table 1. Results of $U$ and $x_0$ inversion.

One could see that ambiguity of turbulent flow retrieval lies in the limit of 15-20%. This value is dependent on the signal-to-noise ratio, which was in our experiments very low. To increase the accuracy of the method we recommend to use noiseless source of turbulent flow, apply methods for increasing sound fluctuations, for example increase velocity of the flow.
CONCLUSION

Experimentally was shown the attempt to retrieve velocity of the turbulent flow and it position in the space using the analysis fluctuation spectra of the acoustical signal of different frequencies propagated along one path. Thus we demonstrated here the principals of single acoustical path tomography, where instead of multiple paths we use multi-frequency acoustical signals. Results of experimental inversion show the more closer to the real measured data occurs the inversion by signals with pairs of the most close frequencies. Fresnel zones dimension differ no much each other and frozen turbulence model conditions fulfills in the best way to determination of cut off frequency.

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