SUBBSPACE TRACKING FOR DOMINANT MODE REJECTION BEAMFORMING

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Abstract: *Dominant mode rejection enables adaptive beamforming when the number of snapshots is limited. This situation typically occurs for the detection of transient signals in passive sonar processing. Moreover, the adaptive beamforming filter needs to be computed at a very high rate to adapt to the processed signal. This task is very demanding in terms of computation cost. However the processing only requires the first eigenvalues and eigenvectors, which may be computed recursively using subspace tracking algorithms. An implementation of a subspace tracking algorithm is here proposed. This implementation is based on previous studies and its numerical stability is tried out both on simulated and real data for dominant mode rejection beamforming. A comparison is finally made between conventional and dominant mode rejection in the context of transient detection.*

Keywords: *Adaptive beamforming, passive sonar, subspace tracking, dominant mode rejection beamforming*

INTRODUCTION

The main goal of passive sonar array processing is to detect low level signals by removing loud interferers and rejecting self-noise. The natural way to achieve this goal is to increase the array size. Yet, loud interferers limit the array gain for the detection of low level source through sidelobes jamming when the beamformer is conventional. On the opposite, sidelobes are kept under noise level when the array is adaptively processed by placing notches in the direction of these interferers. To this end, the adaptive processing, known as the Capon beamforming, computes the beamforming weights using the inverse of the noise covariance matrix. This noise covariance matrix is estimated with the received signal impinging the array, and the number of needed snapshots is linked to the number of sensors through the Capon statistics. To avoid SNR losses, the required number is at least three times the number

of sensors. Consequently, the larger the array is, the more time is needed to estimate the weights. However, at the same time, the data might only be stationary during short period of time, especially for large arrays, fast-changing environments or transient signals. Faster convergence of the adaptive processing is therefore required in these situations. This is achieved through the reduction of the adaptive degrees of freedom. Several techniques are available: subarray [1] or beamspace processing are the most common ones. Data dependent rank reduction is also popular and considered more optimal for rejection: krylov subspaces, or eigenspaces beamformer are the best examples. The dominant mode beamformer uses the dominant eigenvectors of the (badly) estimated covariance matrix, and use them to construct a better estimate by constructing the unknown minoring eigenspace as an uncorrelated noise, whom level is modelled or estimated.
Along with the need of rank reduction, adaptive filter weights computation is needed more often for fast-changing environment. Ultimately, this computation is processed at each new snapshot to adapt to very short transient signals. It might also serve as a basis for audio processing prior to inverse Fourier transform. To this end, minimizing computing costs are reached by using subspace tracking algorithms, which only calculate the dominant eigenvectors. The bottleneck of such recursive algorithms is the numerical stability. A fast and stable implementation is here used, and slightly modified to achieve exact equivalence with a direct method, that computes all the eigenvectors and eigenvalues of the sensors covariance matrix.

This paper is organized as follows. The first part presents the DMR beamformer, while the second part details the subspace tracking algorithm. Its numerical stability and precision are shown on sea signals, and the benefits over a conventional beamforming are discussed.

DOMINANT MODE REJECTION ADAPTIVE BEAMFORMER

The adaptive beamforming aims at maximising the output SNR beam, by minimizing the output power while maintaining the signal contribution in the steering direction:

$$\mathbf{h} = \arg\min \mathbf{h}^{+}\boldsymbol{\Gamma}\mathbf{h} \;\; s.t. \;\; \mathbf{h}^{+}\mathbf{d} = 1 \tag{1}$$

The solution to this optimization problem is the usual Capon filter:

$$\mathbf{h} = \frac{\boldsymbol{\Gamma}^{-1}\mathbf{d}}{\mathbf{d}^{+}\boldsymbol{\Gamma}^{-1}\mathbf{d}} \tag{2}$$

When the number of snapshots N is greater than the number of sensors n, the covariance matrix is estimated with the snapshots of the received signal vector $\mathbf{x}_i$:

$$\boldsymbol{\Gamma}_{\mathbf{SMI}} = \hat{\boldsymbol{\Gamma}} = \frac{1}{K}\sum_{i=1}^{K} \mathbf{x}_i\mathbf{x}_i^{+}, K > n \tag{3}$$

When this sample matrix inversion (SMI) is not feasible due to the snapshot deficiency or would involve high signal losses (K < 3n), the covariance matrix is estimated by using the r dominant eigenvectors $\mathbf{U}_s$ and eigenvalues $\boldsymbol{\Lambda}_s = \mathrm{diag}([\lambda_i]_{i \le r})$ of $\hat{\boldsymbol{\Gamma}}$:

$$\boldsymbol{\Gamma}_{DMR} = \mathbf{U}_s[\boldsymbol{\Lambda}_s - \sigma^2 \,\mathbf{I}_r]\,\mathbf{U}_s{}^{+} + \sigma^2\,(\mathbf{I}_K - \mathbf{U}_s\mathbf{U}_s^{+}) \tag{4}$$

The number of dominant eigenvectors depends on the number of available snapshots and, as for Capon beamforming, this number of freedom degrees r should respect K > 2 r. The level of uncorrelated noise is either known (electrical noise) or estimated as the mean of lowest eigenvalues of the covariance matrix (possibly estimated using a greater number of snapshots). The adaptive filter is then computed using this constructed covariance matrix:

$$\mathbf{h_{DMR}} = \frac{\mathbf{\Gamma}_{DMR}^{-1}\mathbf{d}}{\mathbf{d}^{+}\mathbf{\Gamma}_{DMR}^{-1}\mathbf{d}} \tag{5}$$

The output of the DMR filter $\mathbf{w_{DMR}^{+}x}_i$ is finally written as follows:

$$\mathbf{h_{DMR}^{+}x}_i = \frac{\mathrm{d}^{+}\mathbf{U}_s(\mathbf{\Lambda}_s - \sigma^2\,\mathbf{I}_r)\mathbf{U}_s{}^{+}\mathbf{x}_i + \sigma^2\left(\mathbf{d}^{+}\mathbf{x}_i - \mathrm{d}^{+}\mathbf{U}_s\mathbf{U}_s{}^{+}\mathbf{x}_i\right)}{\mathrm{d}^{+}\mathbf{U}_s[\mathbf{\Lambda}_s - \sigma^2\,\mathbf{I}_r]\,\mathbf{U}_s{}^{+}\mathbf{d} + \sigma^2\left(n - \mathrm{d}^{+}\mathbf{U}_s\mathbf{U}_s{}^{+}\mathbf{d}\right)} \tag{6}$$

In this last expression, the main computations are the following matrix product: $\mathbf{U}_s{}^{+}\mathbf{x}_i, \mathrm{d}^{+}\mathbf{U}_s, \mathbf{d}^{+}\mathbf{x}_i$. It is also remarkable to notice that only the dominant eigenvectors and eigenvalues are needed to implement the dominant mode rejection beamforming. When the beams need to be computed very often, the snapshots are only partly renewed. In this case, subspace tracking algorithms might be used: new eigenvectors and eigenvalues are computed from the former ones instead of computing them again from the snapshots.

SUBSPACE TRACKING ALGORITHM

The goal of subspace tracking algorithm is to recursively compute an orthogonal projector $\mathbf{\Pi}(t)$ on the r dominant eigenvectors subspace of the rank modification of the covariance matrix:

$$\mathbf{C_{xx}}(t) = \beta\mathbf{C}_{xx}(t-1) + \mathbf{x}(\mathrm{t})\mathbf{x}(\mathrm{t})^{+}, \qquad \mathbf{C_{xx}}(t) \in \mathbb{C}^{n,n} \tag{7}$$

The covariance matrix $\mathbf{C_{xx}}(t)$ is linked to the covariance used in the beamforming by a scaling factor and β is the typical forgetting factor such that $0 < \beta < 1$. The rectangular window can be achieved with two successive rank one modifications.

This projector, $\mathbf{\Pi}(t) = \mathbf{W}(t)\mathbf{W}(t)^{+}$, constructed with the orthogonal basis $\mathbf{W}(t)$, maximises the generalized Rayleigh quotient $J\left(\mathbf{\Pi}(t)\right)$:

$$\mathbf{\Pi}(t) = \arg\max J\left(\mathbf{\Pi}(t)\right), \qquad J\left(\mathbf{\Pi}(t)\right) = \mathrm{Tr}\left(\mathbf{C}_{xx}(t)\mathbf{\Pi}(t)\right) \tag{8}$$

Let $\underline{\mathbf{\Pi}}(t) = \underline{\mathbf{W}}(t)\underline{\mathbf{W}}(t)^{+}$ now be the projector on the subspace at time t-1, augmented with the new vectors generated by the new snapshots. The augmented basis is expressed with the former basis as $\underline{\mathbf{W}}(t) = [\mathbf{W}(t-1)\ \ \mathbf{u}]$, where

$$\mathbf{u} = \mathrm{orth}(\mathbf{e}) \tag{9}$$

$$\mathbf{e} = (\mathbf{I}_K - \mathbf{W}(t-1)\mathbf{W}(t-1)^{+})\,\mathbf{x}(t) = \mathbf{x}(t) - \mathbf{W}(t-1)\mathbf{y}(\mathrm{t}) \tag{10}$$

$$\mathbf{y}(\mathrm{t}) = \mathbf{W}(t-1)^{+}\mathbf{x}(t)$$

The goal is now to find the vector **v** such that, as proposed in [2]:

$$\mathbf{\Pi}(t) = \underline{\mathbf{\Pi}}(t) - \mathbf{v}\mathbf{v}^{+} \tag{11}$$

To achieve the subspace reduction from the augmented basis, this vector $\mathbf{v}$ has to be orthogonal to the new basis. Yet, it is included in the vector space spanned by $\underline{\mathbf{W}}(t)$, and can therefore be expressed as $\mathbf{v} = \underline{\mathbf{W}}(t)\underline{\boldsymbol{\phi}}$. The Rayleigh coefficient is finally:

$$J(\mathbf{\Pi}(t)) = \mathrm{Tr}\left(\underline{\mathbf{C}}_{\mathbf{yy}}(t)\right) - \mathrm{Tr}\left(\underline{\boldsymbol{\phi}}\underline{\mathbf{C}}_{\mathbf{yy}}(t)\underline{\boldsymbol{\phi}}\right) \tag{12}$$
$$\text{with } \underline{\mathbf{C}}_{\mathbf{yy}}(t) = \underline{\mathbf{W}}(t)^{+}\mathbf{C}_{\mathbf{xx}}(t)\underline{\mathbf{W}}(t) = \begin{pmatrix} \beta\mathbf{C}_{\mathbf{yy}}(t-1) + \mathbf{y}(t)\mathbf{y}(t)^{+} & \mathbf{z} \\ \mathbf{z}^{+} & \gamma \end{pmatrix}$$
$$\mathbf{z} = \beta\,\mathbf{W}(t-1)^{+}\mathbf{C}_{xx}(t-1)\mathbf{u} + \mathbf{y}(t)\,\mathbf{u}^{+}\mathbf{x}(t)\mathbf{x}(t)^{+}\mathbf{u}$$
$$\gamma = \beta\mathbf{u}^{+}C_{xx}(t-1)\mathbf{u} + \mathbf{u}^{+}\mathbf{x}(t)\mathbf{x}(t)^{+}\mathbf{u}$$

This Rayleigh coefficient is therefore maximised when $\underline{\boldsymbol{\phi}}$ is chosen as the minoring eigenvectors of $\underline{\mathbf{C}}_{\mathbf{yy}}(t)$. The new (not necessarily orthogonal) basis $\mathbf{T}$ of the dominant eigenspace, linked to the orthogonal projector defined in equation 11, needs to be orthogonal to the vector $\mathbf{v}$. This constraint is only asymptotically respected in [2]. The following basis proposed here respects exactly this constraint:

$$\mathbf{T} = \mathbf{W}(t-1) - \mathbf{u}(\varphi^{+})^{-1}\boldsymbol{\phi}^{+}, \text{ with } \underline{\boldsymbol{\phi}} = \begin{pmatrix} \boldsymbol{\phi} \\ \varphi \end{pmatrix} \tag{13}$$

The update procedure now consists in orthonormalizing $\mathbf{T}$ to get the new basis $\mathbf{W}(t)$. This tricky point is major, as the level of orthogonality is fundamental for the adaptive beamforming. The proposed technique in [2] does not seem to be completely stable [3]. A stabilized version of the YAST algorithm has been proposed in [4], which does not modify the algorithm complexity. On the opposite, a full reorthogonalization increases the complexity of the algorithm to $\mathcal{O}(nr^2)$ instead of $\mathcal{O}(nr)$. However, this choice ensures the numerical stability. It also opens the way to a generalized rank k update, for which all the developments made above are compatible. In the context of beamforming, this generalisation allows for a more flexible choice of the filter update frequency. After all, the cost of full reorthogonalization is virtually similar to the eigenvectors computation cost of $\underline{\mathbf{C}}_{\mathbf{yy}}(t)$ when the rank r is not excessively smaller than the number of sensors.

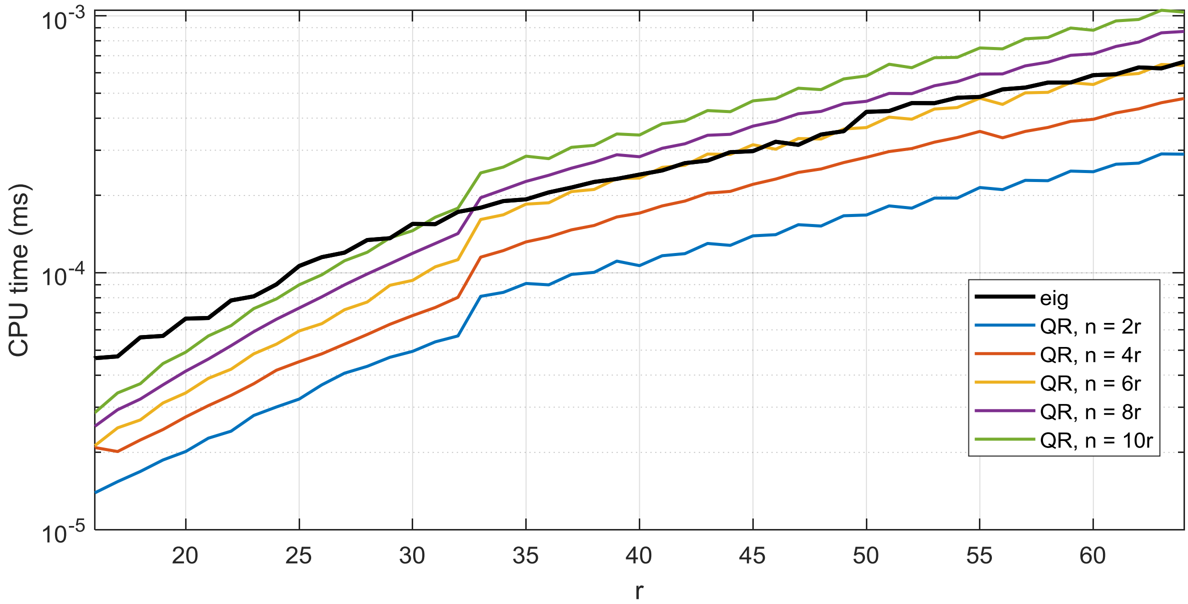

Fig.1: CPU time of orthogonalisation (single precision) and eigenvalue (double precision)

The choices of r and n corresponding to array processing make the cost of full reorthogonalization rather affordable. This full reorthogonalization avoids the risk of numerical instability. The gain on computation cost compared to a direct computation is still very comfortable. Fastest eigenvalues algorithms, such as "divide and conquer", achieve a computational complexity of $\mathcal{O}(n^{2.8})$ for flop counts [5]. Depending on the ratio n/r, the gain on computation costs is theoretically between 7 for n/r = 2 and 630 for n/r = 10.
Numerical stability has been assessed both on simulation and sea signals. As simulations reproduce with difficulty the variety of real signals (marine mammals, sonar emission, self-noise or even electronic glitches), all kind of arrays were processed with this algorithm. No sign of divergence between the recursive and the direct computation were noticed. The processing results from a 32 sensors towed array are presented here with a rank set to 16.

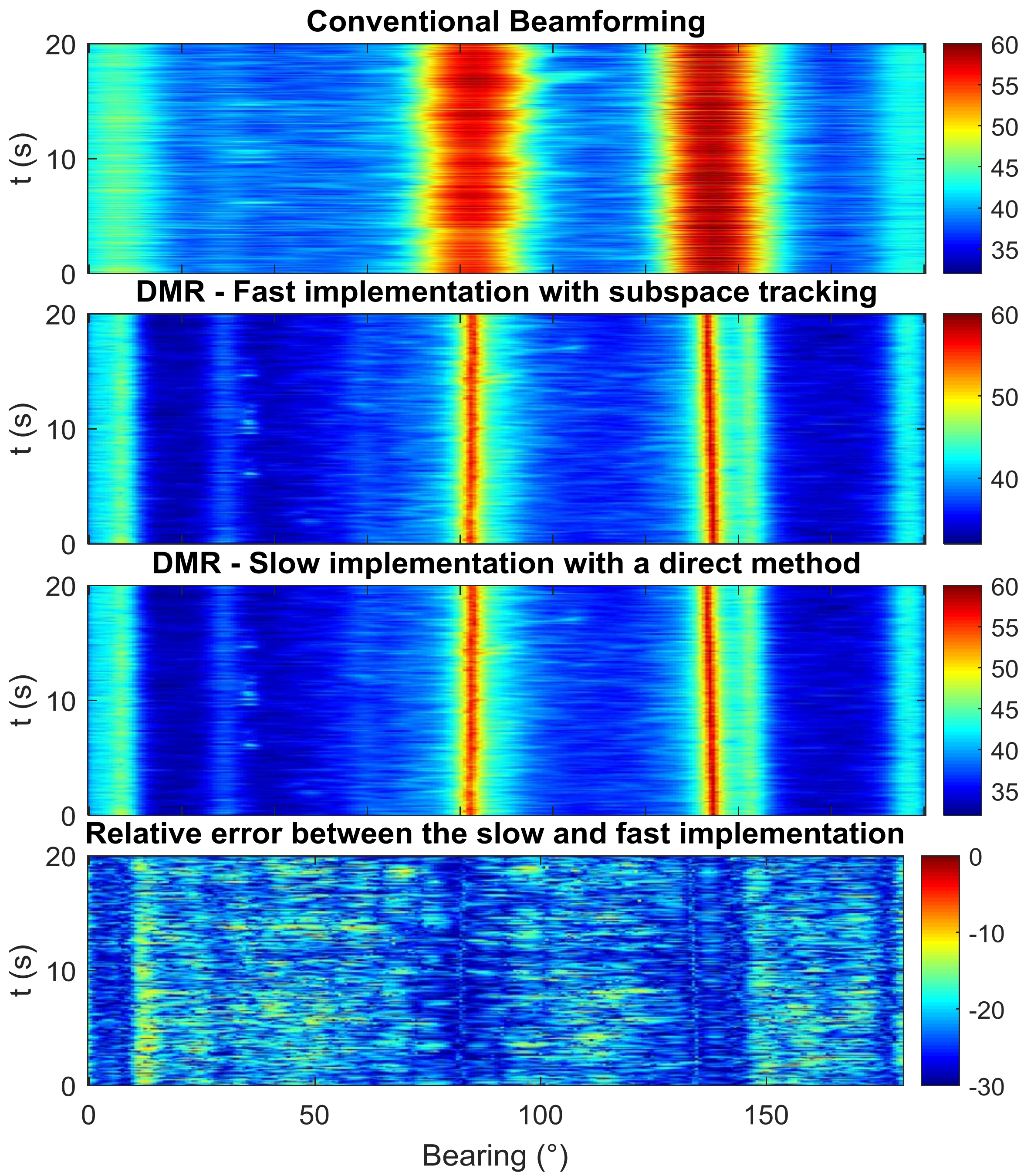

Fig.2: Broadband beams for conventional and DMR beamforming

The snapshots used for filter estimation are collected on 1 second time and 48 Hz band samples weighted by a rectangular window. The chosen rank, 16, respects the rule of thumb that dictates to set the rank, number of adaptive degree of freedom, as the third of the available independent snapshots. This choice made here is consistent with the traditional Capon statistics, requiring the number of snapshots to be three times the number of sensors.
The beams are formed every 125 ms. This very short integration time is adequate for the detection of very short transient signals. Through the proposed algorithm, the transient detection benefit from the loud interferer rejection of the adaptive beamforming, as shown on Fig. 2 at bearing 40°. Indeed, as shown on the last subplot, the fast implementation proposed here does not diverge from the direct implementation. Relative error is higher on the noise than on the sources. Indeed, a fully uncorrelated noise may admit several dominant subspaces.

CONCLUSION

In this paper, a subspace tracking algorithm has been proposed. Compared to previous studies, the proposed algorithm theoretically yields the same subspace as a direct implementation of the rank one modification of the covariance matrix, but with a highly reduced computation cost. A full reorthogonalization at each recursion ensures the numerical stability of the algorithm. For passive sonar applications, the cost of this full reorthogonalization of the basis remains reasonable. It also opens the way for a rank k modification.
This subspace algorithm has been applied to dominant mode rejection beamforming on passive sonar sea signal. Numerical stability is reached, and no divergence has been noticed when comparing the subspace tracking to a direct implementation. This recursive implementation thus emerges as an attractive solution to the dominant mode beamformer for detection of transient signals in the context of passive sonar processing, ensuring the rejection of the adaptive beamformer and the reactivity of a conventional one.

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