AN ESTIMATION ALGORITHM OF DOA FOR VECTOR SENSOR ARRAY BASED ON TENSOR THEORY

Song Li[a,b,c], Zhe Chen[a,b,c], Desen Yang[a,b,c]

[a]Acoustic Science and Technology Laboratory, Harbin Engineering University, Harbin 150001,China;
[b]Key Laboratory of Marine Information Acquisition and Security(Harbin Engineering University),Ministry of Industry and Information Technology; Harbin 150001, China;
[c]College of Underwater Acoustic Engineering, Harbin Engineering University, Harbin 150001, China

SongLi, Room 1103, College of Underwater Acoustic Engineering, Harbin Engineering University, Harbin 150001, China; E-mail: songli@hrbeu.edu.cn

Abstract: *In order to solve the problem that direction of arrival estimate performance of traditional direction of arrival (DOA) estimation algorithm in the case of low signal to noise ratio is degradation. In this paper, a DOA estimation algorithm based on tensor decomposition for vector hydrophone array signals is proposed. The tensor analysis model is introduced to establish the tensor model of the vector hydrophone array signal, the signal subspace is derived by tensor decomposition, which combines the traditional DOA estimation algorithm to estimate the sound source. The DOA estimation algorithm based on tensor decomposition for vector hydrophone array signals in the noise suppression and Signal subspace estimation superior than the traditional DOA estimation algorithm. In simulation, the proposed algorithm is compared with the traditional DOA estimation algorithm, and the results show the effectiveness of the proposed algorithm.*

Keywords: *Tensor decomposition, Vector hydrophone array, Direction of arrival estimation*

1. INTRODUCTION

Nowadays, accurate orientation of the target's orientation is one of the key issues that must be addressed in many future areas. In the underwater acoustic signal processing, in order to determine the direction of arrival (DOA) of the sound wave, the sound field is usually spatially sampled using a vector hydrophone array, and spatial spectrum estimation is performed. Traditional methods of azimuth estimation include subspace algorithms such as MUSIC algorithm and ESPRIT algorithm. The traditional azimuth estimation method has poor performance at low SNR because these algorithms use the particle velocity and sound pressure as parallel data to expand into a vector form, and do not make full use of the sound pressure output from the acoustic vector sensor. The orthogonal relationship between the components of the vibration velocity, therefore, an effective way to solve the above problem is to introduce a tensor operation.

As a unified language for multi-dimensional signal processing, tensor has incomparable advantages for high-dimensional algebraic operations. Compared with matrix or vector, tensor decomposition model is more suitable for multi-dimensional structure of real signal. An important model in tensor decomposition is Tucker tensor decomposition. Tucker tensor decomposition is a high-order singular value decomposition, which focuses on acquiring information feature subspace and retaining the orthogonality of the algorithm.

The current tensor decomposition algorithm is mainly used in image processing. In array signal processing, it is mainly used in electromagnetic vector signals. In the literature, the tensor method is introduced in vector array signal processing, and the tensor is applied in the polarization sensitive array. Combined with the MUSIC algorithm, the vector MUSIC signal algorithm using tensor operation is proposed. The literature introduces the Tucker decomposition model to construct a Kronecker product approximation method as a pre-regulator for image processing. The literature introduces the Tucker tensor decomposition model into motion recognition, and its application extends from static images to dynamic images. The paper proposes a tensor-based real-valued subspace method for estimating the target wavelength direction (DOD) and direction of arrival (DOA) in multiple-input multiple-output (MIMO) radar. In this paper, the received acoustic vector signal data is reconstructed, the tensor model is established, and the corresponding tensor signal subspace is obtained by tensor decomposition, which combines the traditional azimuth estimation method to estimate the sound source. The azimuth estimation algorithm based on tensor decomposition theory proposed in this paper has better noise suppression ability for acoustic vector array signals, which can improve the accuracy of azimuth estimation accuracy.

2. TENSOR DECOMPOSITION THRORY

The n-mode expansion of a tensor is the tensor expanded into a matrix as a column in n-mode. $A_{(n)}(1 \le n \le N)$ is the matrix of n-mode expansion for the tensor $A \in C^{I_1 \times I_2 \times \ldots \times I_N}$,and the following relationship exists between element (i_n, j) and $(i_1, i_2, \ldots, i_N)$:

$$j = 1 + \sum_{\substack{k=1 \\ k \ne n}}^{N} (i_k - 1) J_k , J_k = \prod_{\substack{m=1 \\ m \ne n}}^{k-1} I_m \tag{1}$$

A new tensor is made by $A \in C^{I_1 \times I_2 \times \ldots \times I_N}$ and matrix $U = C^{J_N \times I_N}$

$$B = A \times_n U \in C^{I_1 \times I_2 \times \ldots \times I_{n-1} \times J_n \times I_{n+1} \times \ldots \times I_N} \tag{2}$$

$$b_{i_1, i_2, \ldots, j_n, \ldots, i_N} = \sum_{i_n} a_{i_1, i_2, \ldots, j_n, \ldots, i_N} u_{j_n i_n} \tag{3}$$

3. ACOUSTIC VECTOR SENSOR ARRAY TENSOR MODEL

It is assumed that the signal satisfies the narrow-band far-field plane wave condition, and the incident signals are statistically independent. The background noise received by each array element is Gaussian white noise, and the correlated time radius of the noise is smaller than the time interval of data acquisition. The background noise received by each array element is independent.

There are *k* signals satisfying the conditions of the narrow-band far-field plane wave incident on an equidistant linear array composed of M vector sensors. The incident angle is (θ_k, φ_k)， *k*=1,2,…， *K*， and θ_k, φ_k are the azimuth angle and pitch angle of the kth signal. The number of array elements of the linear array is greater than the number of sound source signals。

If the $a(\Theta_k) \circ u_k$ can be looked as a slice, then the direction tensor $A(\Theta)$ will be got.

$$A(:,:,k)=\begin{bmatrix} 1 & \cos\theta_k\cos\varphi_k & \sin\theta_k\cos\varphi_k & \sin\varphi_k \\ e^{-j\frac{2\pi}{\lambda}d\sin\varphi_k} & e^{-j\frac{2\pi}{\lambda}d\sin\varphi_k}\cdot\cos\theta_k\cos\varphi_k & e^{-j\frac{2\pi}{\lambda}d\sin\varphi_k}\cdot\sin\theta_k\cos\varphi_k & e^{-j\frac{2\pi}{\lambda}d\sin\varphi_k}\cdot\sin\varphi_k \\ \vdots & \vdots & \vdots & \vdots \\ e^{-j\frac{2\pi}{\lambda}d(M-1)\sin\varphi_k} & e^{-j\frac{2\pi}{\lambda}d(M-1)\sin\varphi_k}\cdot\cos\theta_k\cos\varphi_k & e^{-j\frac{2\pi}{\lambda}d(M-1)\sin\varphi_k}\cdot\sin\theta_k\cos\varphi_k & e^{-j\frac{2\pi}{\lambda}d(M-1)\sin\varphi_k}\cdot\sin\varphi_k \end{bmatrix} \quad (4)$$

$A(:,:,k)$ is a slice of the tensor $A(\Theta)$ at the 3th dimensional, and k=1,2,…， K. then $A(:,:,k) = A_K$ is defined. A third-order tensor output model is constructed under L snapshots，

$$X = A \times_3 S^T + N \quad (5)$$

In this formula, $\times_3$ is represented product of tensor and the 3-modulus matrix, and $A \in C^{M\times 4\times K}$， $X \in C^{M\times 4\times L}$， $N \in C^{M\times 4\times L}$.

Obtained in the measurement tensor model output X high order tensor Singular Value Decomposition

$$X = S \times_1 U_1 \times_2 U_2 \times_3 U_3 \quad (6)$$

$S \in C^{M\times 4\times L}$ is the core tensor of tensor X under singular value decomposition, and $U_1 \in C^{M\times M}$ is the left singular vector matrix of tensor X under singular value decomposition in 1-mode. The formula below is satisfied,

$$X_{(1)} = U_1 \cdot \Sigma_1 \cdot V_1^H \quad (7)$$

At the same theory, $U_2 \in C^{4\times 4}$ is the left singular vector matrix of tensor X under singular value decomposition in 2-mode, and $X_{(2)} = U_2 \cdot \Sigma_2 \cdot V_2^H$ is satisfied; $U_3 \in C^{L\times L}$ is the left singular vector matrix of tensor X under singular value decomposition in 2-mode, and $X_{(3)} = U_3 \cdot \Sigma_3 \cdot V_3^H$ is satisfied.

The we cut off the left singular matrix of tensor X under singular value decomposition in n-mode, and $U_{1S} \in C^{M\times K}, U_{2S} \in C^{4\times K}, U_{3S} \in C^{L\times K}$ are obtained which are made of the column vectors corresponding to K larger singular values are chosen. Then $U_{1N} \in C^{M\times(M-K)}, U_{2N} \in C^{4\times(4-K)}, U_{3N} \in C^{L\times(L-K)}$ are obtained by the other columns.

So we can have that

$$X_{(1)} \approx U_{1S} \cdot \Sigma_{1S} \cdot V_{1S}^H \tag{8}$$

$$X_{(2)} \approx U_{2S} \cdot \Sigma_{2S} \cdot V_{2S}^H \tag{9}$$

$$X_{(3)} \approx U_{3S} \cdot \Sigma_{3S} \cdot V_{3S}^H \tag{10}$$

In this formula, $\Sigma_{ns}(n=1,2,3)$ are the diagonal matrix consisting of K singular values of the n-mode expansion of the tensor X, respectively.

$$X = S \times_1 U_{1S} \times_2 U_{2S} \times_3 U_{3S} \tag{11}$$

So the core tensor can be defined as

$$S_S = X \times_1 U_{1S}^H \times_2 U_{2S}^H \times_3 U_{3S}^H \tag{12}$$

Which is

$$X = S_S \times_1 U_{1S} \times_2 U_{2S} \times_3 U_{3S} \tag{13}$$

We define the tensor signal subspace as

$$U_S = S_S \times_1 U_{1S} \times_2 U_{2S} \tag{14}$$

Since the tensor model steering matrix is

$$A_K = a(\theta_k) \circ u_k \tag{15}$$

We could define the following formula at the tensor mode, which are 1-mode Signal subspace is U_{1S}, 2-mode Signal subspace is U_{2S}, and 1-mode noise subspace is U_{1N}, 2-mode noise subspace is U_{2N}:

$$U_{1S}U_{1S}^H = \text{span}(a(\theta_1), a(\theta_2), \cdots, a(\theta_k)) \tag{16}$$

$$U_{1S}U_{1S}^H = \text{span}(u_1, u_2, \cdots, u_k) \tag{17}$$

So we can know that

$$U_{1N}U_{1N}^H \cdot a(\theta_k) = 0 \tag{18}$$

$$U_{2N}U_{2N}^H \cdot u_k = 0 \tag{19}$$

According to the tensor model, the relationship between the steering matrix and the noise subspace can be expressed as:

$$A_K \times_1 U_{1N}U_{1N}^H = 0 \tag{20}$$

$$A_K \times_2 U_{2N}U_{2N}^H = 0 \tag{21}$$

It can be seen that the column vector of the tensor model array response matrix is orthogonal to the 1-mode noise subspace, and the row vector is orthogonal to the 2-mode noise subspace, which is called dual mode orthogonality.

So the spectral estimation formula based on tensor decomposition azimuth estimation algorithm is obtained.

$$P_{MUSIC}(\theta,\varphi) = \frac{1}{\left\| A \times_1 U_{1N}U_{1N}^H \times_2 U_{2N}U_{2N}^H \right\|} \tag{22}$$

4. SIMULATION ANALYSIS

In this simulation analysis, the acoustic vector sensor array uses a half-wavelength equidistant linear vertical array. By comparing the effectiveness of the traditional MUSIC algorithm and the performance verification experiment based on the tensor decomposition azimuth estimation algorithm. And condition of the simulation are: the number of elements M=8， the distance between the neighboring elements d=λ/2, the azimuth angle and pitch angle both are 45°, the centre frequency is 300Hz， the snapshot L=1024， the background is White Gaussian Noise and SNR is -5dB。

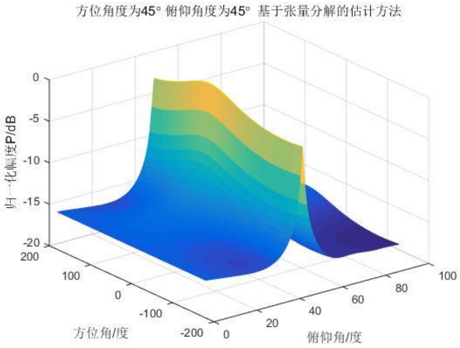

Fig.1: Three-dimensional simulation diagram based on MUSIC algorithm

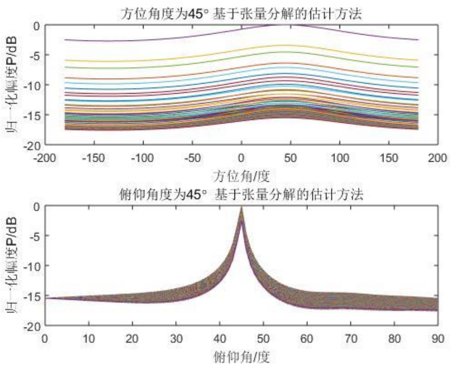

Fig.2: Simulation diagram based on MUSIC algorithm

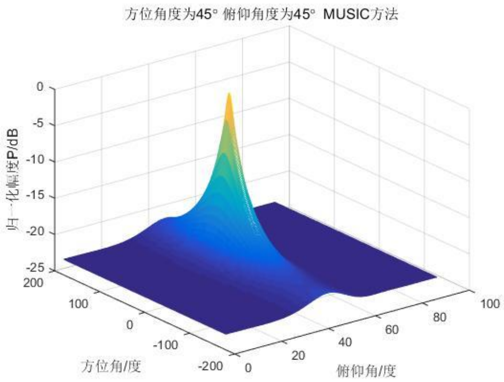

Fig.3: Three-dimensional simulation diagram based on tensor decomposition azimuth estimation algorithm

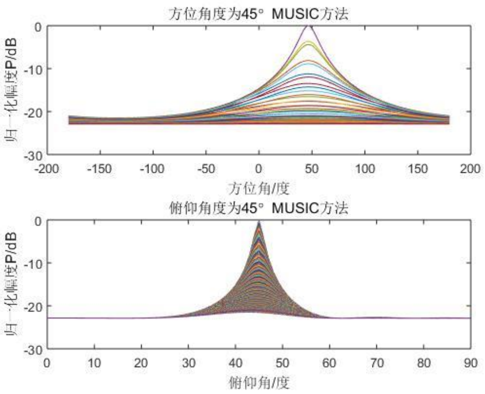

Fig.4: Simulation diagram based on tensor decomposition azimuth estimation algorithm

Figure 1 and Figure 2 are simulation diagrams of the MUSIC algorithm. It can be seen from the figure that the azimuth estimation performance is poor at low SNR. Figures 3 and 4 are simulation diagrams based on the tensor decomposition orientation estimation algorithm. It can be seen that under the same simulation conditions, the tensor decomposition based orientation estimation algorithm has lower side lobes, higher spectral peaks, and the estimation accuracy is greatly improved. Therefore, the proposed algorithm has great engineering application value.

5. CONCLUTION

The traditional azimuth estimation method has certain errors in estimating the signal subspace and the noise subspace, resulting in the accuracy of the estimated sound source incident angle is not high. A tensor decomposition based azimuth estimation algorithm proposed in this paper can better suppress noise due to high-order singular value decomposition, and has better noise suppression ability for sound vector array signals, which can improve low signal-to-noise ratio. The performance of the time orientation estimate.

6. ACKNOWLEDGEMENTS

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