UNCERTAINTY ANALYSIS IN SONAR PERFORMANCE MODELLING BY LATIN HYPERCUBE SAMPLING

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Abstract: *We have studied the possibility to use stratified sampling for uncertainty analysis applied to underwater applications. The method is called Latin Hypercube Sampling (LHS) and the idea behind this approach is to sample only a representative part of the probability distributions so that the number of runs can be reduced significantly. Usually, the number of runs are in the order of 50-100 compared to traditional Monte Carlo sampling where you have millions of runs. An important issue is the difference between uncertainty and sensitivity analysis, which we demonstrate with an active acoustic scenario. Thereafter, we show how we can use LHS in an area where you have quite large uncertainties in the geo-acoustic properties. We have identified the need of presenting the uncertainties to the user in an easy understandable way in a single plot. A new visualization parameter is defined, TPOD (Total Probability of Detection), to meet this requirement. Our studies using LHS show that it is easy to apply and give useful insight in complex uncertainty dependencies. However, for relevant output we need to have good knowledge about the individual parameter probability distributions, both regarding absolute values and representative distributions.*

Keywords: *Sonar, performance calculations, uncertainty analysis, Latin Hypercube Sampling*

INTRODUCTION

Many years of research in underwater acoustics, together with development of fast computers and efficient algorithms have resulted in that we today can do truthful simulations of our sensor systems, and do performance predictions with high precision.

However, good quality predictions are only possible if the input data is well known, i.e. the errors in the parameters are small. Unfortunately this is not always the case, some parameters such as the geo-acoustic bottom and sub-bottom parameters are sometimes not well known.

The calculations you normally do in performance predictions show the probability of detection (POD) for a given set of parameters, as a function of distance and depth, giving the detection distance for your sonar, or the counter-detection distance for a possible threat. Now, due to uncertainties in the input parameters, this value need not be the most representative. However, it can often be sufficient to just know this value, for example when setting the optimal depth for a variable depth sonar (VDS). But in some situations we also need to know the uncertainty in the calculations. For example, a submarine exposed to ASW wants to know if the hunting ship has a chance to detect it or not, in order to be able to navigate safely or to be able to take calculated risks to escape the situation. Then it is important to know not only the most probable detection distance, but also the minimum and maximum distances.

Uncertainties in complex calculations, especially those using nonlinear models, are not only dependent on individual input errors, but also on how parameters in combination affect the end result. We have therefore studied how to calculate these uncertainties correctly.

To define the concept of *uncertainty* analysis, we compare it with the concept of *sensitivity* analysis. As described in [1] *sensitivity* analysis is performed by perturbing one parameter, while keeping all other parameters fixed. On the other hand, in *uncertainty* analysis several parameters are varied at the same time and in this way the entire output space can be studied. One can say that the sensitivity analysis is local, while the uncertainty analysis takes a global perspective.

As also pointed out in [1] there are three main areas regarding uncertainties; parameter uncertainties, structural uncertainties (model errors) and stochastic uncertainties. The two former are mainly due to the lack of knowledge regarding parameters or how physical processes are simplified in the model used. Such uncertainties can be reduced with better knowledge of parameter values and better physical approximations in the model. Stochastic uncertainties or, in other words, the variability of natural phenomena cannot be eliminated. Through increased knowledge, however, one can get a better understanding of these uncertainties. In this report, we have primarily studied the influence of parameter uncertainties.

LATIN HYPERCUBE SAMPLING

The basic approach behind Latin Hypercube Sampling (LHS) is to divide the probability distributions into a number of bins, each with equal probability, where the number of bins is an arbitrary value set by the user. This is called stratification. Each stratification can only be used once when combining the parameters included in the uncertainty analysis which means that the number of runs will be equal to the stratification value [2].

Given the stratification value the idea is to compute a large enough number of parameter combinations to cover the most important uncertainty features (Fig.1). After that the wave propagation model is run for every combination computed by LHS. The advantage of LHS is that the number of dispersion calculations is significantly reduced compared to Monte-Carlo methods (MCM). In LHS, we typically talk about 50-100 runs while in brute force MCM the number are millions. The selection of the stratification number is dependent on the specific global uncertainty analysis, and typically, with an increased number of parameters with uncertainties the more stratifications you need. Many times you have to study the specific problem in order to select the proper stratification number.

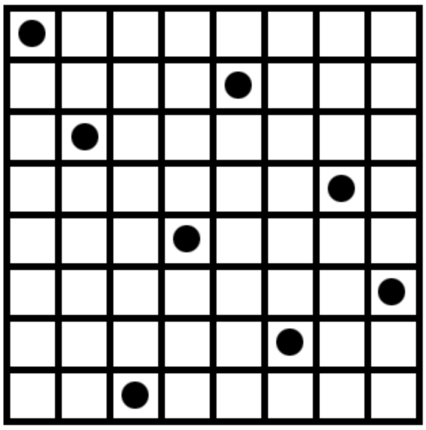

Fig.1: A two-parameter problem with a stratification of 8. The total number of combinations are 64 (as required for a Monte-Carlo simulation). However, the LHS method generates 8 evenly distributed combinations (black dots).

Both LHS and MCM requires knowledge of the parameter uncertainties and their probability distributions. These distributions depend on the nature of the physical problem. In this project we have assumed that the parameter distribution is normal.

UNCERTAINTY ANALYSIS COMPARED TO SENSITIVITY ANALYSIS

In order to gain an understanding of the uncertainties in the result, it is useful to supplement the uncertainty analysis with a sensitivity analysis. To illustrate this, we present results from an uncertainty analysis where we imagine that we have an active VDS sonar searching for a submarine. We study uncertainties in three parameters: the VDS dome depth, the ambient noise, and Lambert's backscattering coefficient. We use a ray-trace model (MultiMoc) for sonar performance calculations, and LHS is used to generate global uncertainty. This is then compared to runs where we fix all parameters except the one that is varied.

Normally, results from sonar performance calculations are displayed as POD. It is calculated based on the assumption that the detection probability follows the so-called ROC curve (Receiver Operating Characteristic). Now, when we have an ensemble of calculations, we have chosen to visualize the result as the "total" POD, *TPOD*, which we also think is the simplest presentation method. If the TPOD is 90%, it means that 90% of the individual runs in the LHS analysis have a detection probability over a given threshold. The threshold we have used in our studies is 50%, which is normally used in performance calculations for surveillance sonar. Thus, if the TPOD is 10%, it means that 10% of the individual runs in the LHS analysis have a detection probability above the 50% threshold. Thus we take into account both favorable and poorer propagation conditions.

In the uncertainty analysis, the parameters have been assigned a normal distribution which is described by an average value and a standard deviation. Regarding Lambert's back scattering coefficient, a fairly large range is chosen which extends from hard rock bottom to sand. Figure 2 shows the result from calculations in a typical summer situation in the Baltic, with a well-developed thermocline at about 20 m.

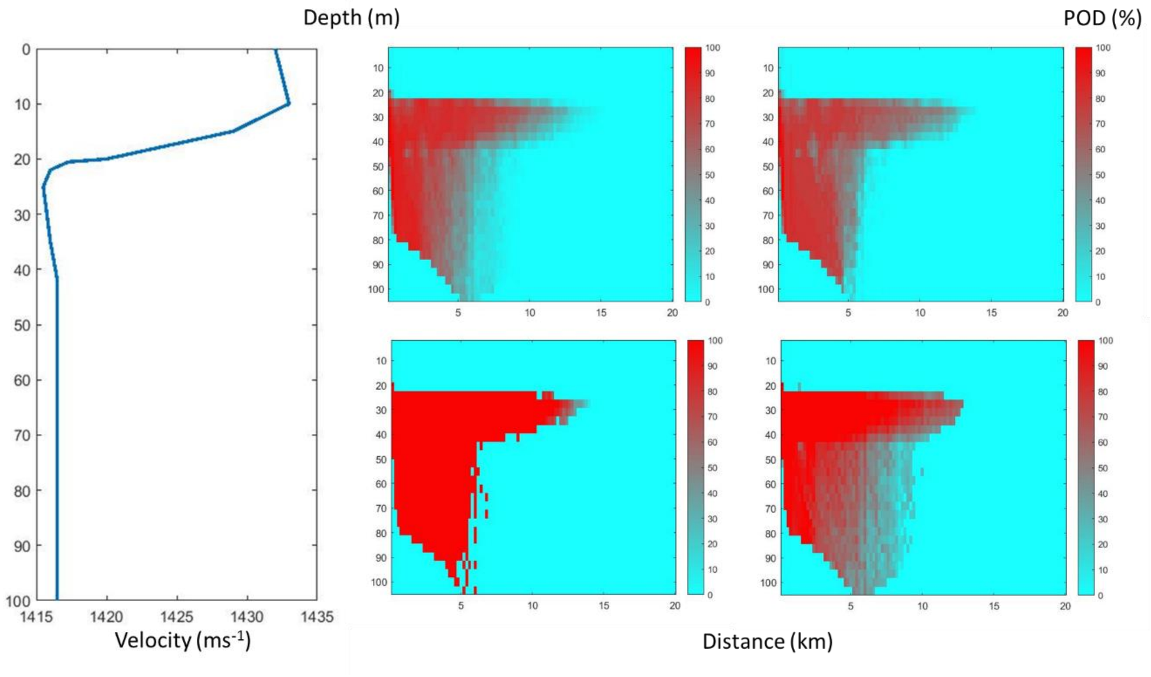

Fig. 2: Detection distance (TPOD) for the uncertainty analysis (top left), sensitivity to ambient noise (bottom left), sensitivity to transmitter depth (top right) and sensitivity to Lambert's coefficient (bottom right).

The TPOD from the uncertainty analysis (Fig.2, top left) shows that to be really sure of detection, the target should be within 2-3 km from the transmitter and in the sound channel up to 6 km. In some of the runs, detection can take place in longer distances, out to 6-12 km. The transition zone between always discovered to rarely detected is approximately 3-5 km. However, this says nothing about which parameter combinations give short and long ranges, respectively. Therefore, there is a need to make a comparison with a sensitivity analysis.

First, we keep Lambert's coefficient and the transmitter depth constant, and the ambient noise varies linearly. In this case, there is very little sensitivity to the ambient noise (Fig.2, bottom left). When we vary the ambient noise there is no variation in detection distances, and the transition zone between always detected to not-detected is short. In addition, in a normal surveillance situation, it is quite easy to measure the ambient noise with sufficient accuracy. The global uncertainty therefore depends very little on this parameter.

When the transmission depth is varied a greater effect is obtained (Fig. 2, top right). This image begins to resemble the global uncertainty analysis (top left) and it tells us that the distance of detection depends strongly on the transmitter depth, which is a typical behavior in shallow waters.

Lastly, we vary Lambert's coefficient, which also has a large impact on the detection distance (Figure 2, bottom right). The detection distance is strongly dependent on the bottom characteristics.

All in all, we can conclude that this is a reverberation-limited case where we also have a sensitivity to the transmitter depth, depending on the sound velocity curve. In order to get a complete picture of how the performance calculations depend on the input parameters, it is helpful to have both a global uncertainty analysis complemented with a sensitivity study. Thus, from a tactical perspective, it would be desirable if a combination of these calculations can be visualized. One way to visualize the results is demonstrated in the example below.

EXAMPLE: SAFE COUNTER-DETECTION

In our example a submarine is out on an exploration mission in a shallow area which is guarded by bottom-mounted passive surveillance sonar. The question now is how close we dare to go without risk of discovery. We know the positions and the capability of the systems relatively well, so we can calculate their detection distances provided we also have correct values for environmental parameters. We assume that the sound velocity in the water and the ambient noise is known. However the geo-acoustic parameters of the sediment layers is not well known. As the performance of a passive low-frequency sonar in shallow water greatly depends on the bottom characteristics, we need to make some assumptions regarding the sediment parameter values, and the possible errors in those assumptions. The best thing we can do is to assume that the area is similar to another known area with a similar geological structure. The assumed parameter values are given in table 1.

Parameter	Mean value	Standard deviation
Sound speed	1480 m/s	100 m/s
Density	1400 kg/m^3	50 kg/m^3
Absorption	0.2 dB/λ	0.1 dB/λ
Layer thickness	50 m	40 m

Table 1: Parameter values for LHS-calculations.

In this case, the calculations show that up to 3 km from the sensor there is a great risk of detection, but if one is to be absolutely sure of passing unnoticed it is necessary to pass at least 6 km distance, given the uncertainties estimated.

A sensitivity analysis has also been carried out where the sound velocity in the sediment layer has been varied within a given interval, and all other parameters are set to the mean value (table 1). Fig. 3 shows the mean value (black-solid line) and the standard deviation (black-dotted line) plotted from this sensitivity analysis for detection distances. The lines illustrate that part of the variations in detection distances is captured by the sensitivity analysis.

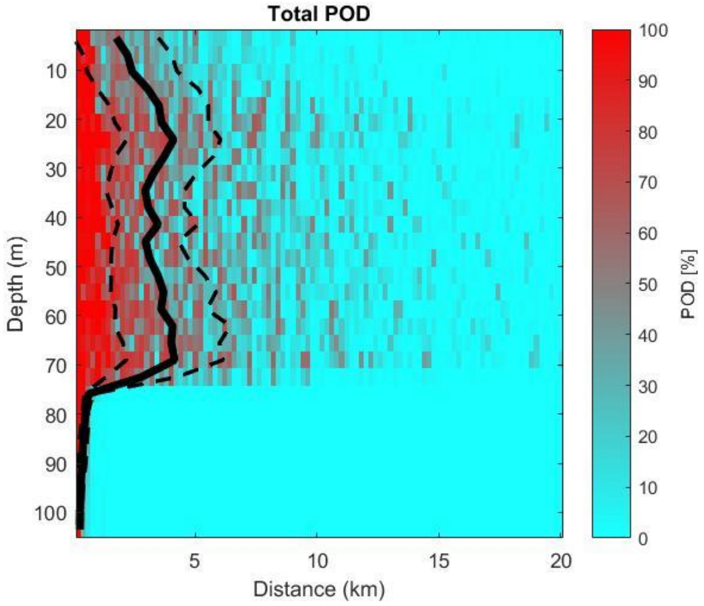

Fig. 3: The TPOD for detection (blue and red colours) by a qualified sonar compared to a sensitivity analysis where only the sound velocity profile is varied: Mean value (solid black line) and standard deviations (dotted lines).

We find that the TPOD becomes an effective way of analyzing the detection distance from an uncertainty perspective. Instead of analyzing 50 different POD plots, we can visualize the same information in a single figure. These calculations are not particularly heavy, and it is fully realistic to introduce these in tactical support systems.

CONCLUSIONS

Our study of the LHS method shows that it is very useful for understanding how uncertainties in input data affect complicated propagation calculations. In our example, we show important aspects such as input uncertainties and its distributions, as well as the importance of choosing the right stratification. Practical experience shows that a suitable stratification for LHS is somewhere between 50 - 100.

However, the uncertainty analysis will depend on good estimates of the parameter uncertainties, both regarding size and what the distribution looks like. In these studies, we have made the assumption that parameter values are normally distributed, but studies are needed to investigate if this really is the case.

In our example, we show how large the variations in detection distance predictions can be, with reasonable values of input data. In an ordinary single calculation of the detection distance, the uncertainty in the result makes it necessary to address safety margins. A calculation with LHS gives all possible outcomes, so the result can be used with greater confidence. We believe that it would be beneficial to introduce LHS into our operational tactical support systems. Here, we propose that the concept of TPOD is introduced, which is an effective way of visualizing the uncertainty in the calculations.

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