# TRANSMISSION PATTERN OPTIMIZATION MODEL FOR CONFORMAL CONTOUR DESIGN

Christoph Zimmer[a], Nils Theuerkauf[b], Dieter Kraus[c], Anton Kummert[d]

[a] Bergische Universität Wuppertal, Rainer Gruenter Str. 21 42119 Wuppertal, Germany
[b]ATLAS ELEKTRONIK GmbH, Sebaldsbrücker Heerstr 235, 28309 Bremen, Germany
[c] Hochschule Bremen, Neustadswall 30, 28199 Bremen, Germany
[d] Bergische Universität Wuppertal, Rainer Gruenter Str. 21 42119 Wuppertal, Germany

Contact: Christoph.Zimmer@atlas-elektronik.com

**Abstract:** *To achieve desired transmission or reception characteristics either optimized amplitude and phase shadings are assigned to each transducer of a linear array or a conformal transducer placement is applied avoiding the necessity of individually driven channels. For higher frequency applications a desired characteristic can also be obtained by conformal shaping of a single transducer.*

*In order to design a conformal transducer with differing horizontal and vertical requirements and a large bandwidth a three-dimensional contour needs to be optimized using numerical optimization techniques. To reduce manufacturing and curvature constraints the resulting contour is milled onto a single composite ceramic stack which also results in the ceramics polarization direction being no longer orthogonal to the conformal surface. To model the resulting acoustical behavior at moderate complexity we chose to adapt a discrete source model by attaching an inherent element characteristic which is differing for each source as a result of the local gradient for the surface contour. Our approach is to apply a linear combination of two characteristics- one along the main vibration direction and hence in main direction of the ceramic and the second orthogonal on the conformal surface to obtain a realistic overall result. These characteristics are acquired based on exemplary FEM simulations using simplified two-dimensional setups for a limited set of sloping angles.*

*The optimization results are verified by measurements obtained with two differing transducers manufactured using the optimized contours. With only minor deviations from the simulation results we present a promising method for conformal contour optimization which can also be applied easily to other applications, element sizes or desired transmission characteristics if required.*

**Keywords:** *Conformal Transducer, Optimization, Beampattern Calculation*

## 1. INTRODUCTION

Desired transducer or antenna characteristics can be achieved by a variety of methods where either element positions which are typically obtained by applying numerical optimization techniques which require a sufficient transmission pattern model. Optimum drive- or arrangement calculation by local [1]-[2], or global [3] numerical optimization techniques as well as their comparison [4] has been of major interest in the last decades till recent applying the well-known point source model [5]. If a constant characteristic is required the beam shaping is preferably achieved by conformal placement of the transducers avoiding the necessity of separated- and individually driven elements.

In this work we apply numerical optimization to design the conformal contour of a relatively small single transducer to achieve a desired transmission characteristic over a large frequency domain. To reduce manufacturing and curvature constraints the resulting contour is milled onto a single composite ceramic stack which also results in the ceramics polarization direction being no longer orthogonal to the conformal surface which is considered by applying a sophisticated element characteristic. Two contours are optimized for differing requirements and the design is validated by comparing simulation results to test measurements.

## 2. CONTOUR SPECIFICATION

For this design we describe the contour of the transducer by a polynomial which in general allows every continuous shape and hence a problem specific solution. The contour height $y$ is given by

$$y(x) = \sum_{i=1}^{N} p_i x^i . \tag{1}$$

where $x$ is the length coordinate centered around $x = 0$ and $p_i$ are the polynomial coefficients which are the parameters to be optimized. For this work we consider only symmetric beampattern requirements and hence all odd $p_i$ are equal to zero. Nevertheless, also asymmetric contours are possible which also results in a more complex optimization due to the increased number of optimization parameters. The order $N$ of the polynomial is chosen as tradeoff between shape synthesizability and optimization complexity and hence is problem specific.

Nevertheless, the flexibility of the polynomial description also allows undesired contours which e.g. cannot be manufactured or do not fit secondary requirements. Hence a constrained optimization is applied further specified in Section 5.

## 3. TRANSDUCER TRANSMISSION PATTERN CALCULATION

With the aim to design the beampattern of a single element the success is mainly depending on the accuracy of the applied model. Moreover, the model should also cause low computation costs if included into a numerical optimization routine. Hence, we apply the discrete source model described in [5] where the obtained contour is sampled equidistantly with in total $N$ sources and extend it by applying a special element characteristic $c(\varphi, f)$. For a later comparison with tank experiments the transmission characteristic is calculated range dependent, where the complex beampattern at elevation $\theta = 0°$ is given by

$$b(\varphi, f \mid r) = \gamma \sum_{n=1}^{N} \frac{1}{r_n(\varphi \mid r)} c(\varphi, f) e^{jkr_n(\varphi \mid r)} . \tag{2}$$

Here $\gamma$ is a normalization parameter, $k = 2\pi f / c$ denotes the wavenumber, $r$ and $\varphi$ are the polar coordinates of the test probe measured relative to the element center and $r_n$ represents the distance between the $n$-th point source on the surface of the element and the test probe. The logarithm of the squared magnitude of $b(\varphi, f \mid r)$ is called beampattern.

$$B(\varphi, f \mid r) = 10\log_{10}\left(\left|b(\varphi, f \mid r)\right|^2\right) \tag{3}$$

The conducted test measurements motivated a more sophisticated modeling of the element characteristic. Due to the chosen design in which the ceramic rods are no longer normal to the transducer surface, we model the element characteristic by a linear combination of a forward component $c_f(\varphi, f)$ and a surface normal component $c_n(\varphi, f)$, i.e.

$$c(\varphi, f) = \alpha c_f(\varphi, f) + (1-\alpha) c_n(\varphi, f) . \tag{4}$$

The design parameter $\alpha$ as well as the characteristics $c_f$ and $c_n$ are chosen to match exemplary FEM simulation results obtained with simplified two-dimensional models.

## 4. NUMERICAL OPTIMIZATION

The applicability of a contour is evaluated by comparing the resulting beampattern $B(\varphi, f \mid r)$ to a desired beampattern $\hat{B}(\varphi, f)$. As cost function the magnitude squared deviation is applied, i.e.

$$c = \int_b \int_{\varphi_s} \left| B(\varphi, f \mid r) - \hat{B}(\varphi, f) \right| d\varphi \, df \tag{5}$$

where $\varphi_s$ is the desired operating sector for the transducer and $b$ the corresponding bandwidth. The cost function is minimized using the MATLAB optimization toolbox. Furthermore linear constraints are added to control the maximum contour height, maximum sloping and ensure the convexity of the obtained solution.

For this work we depict two example contours in Fig. 1 which are obtained by numerical optimizations for different specifications. ‚Contour 1„ should provide a constant opening angle with an approximately parabolic main lobe. ‚Contour 2„ should achieve a flat and low ripple operating sector of 30° in total with least possible sensitivity elsewhere. The contour lengths were selected due to secondary specifications.

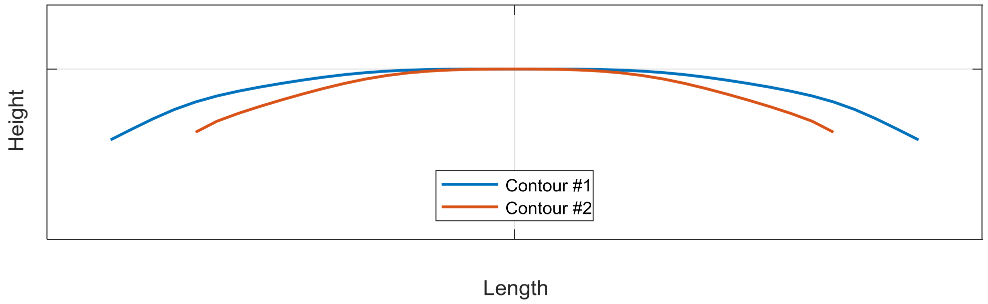

*Fig.1: ‚Contour 1’ and ‚Contour 2’ obtained applying numerical optimization for two different design goals*

## 5. MEASUREMENT EVALUATION

The simulated and measured beampattern alongside with the desired beampattern shape is shown in Fig. 2 for ‚Contour 1„ and in Fig. 3 for ‚Contour 2„ for a set of six frequencies.

For ‚Contour 1„ in Fig. 2 the approximation of the cosine shaped desired pattern is least optimal for the lowest frequency due to the low effect of the curvature on comparably low frequencies. The approximation achieves good results for the frequencies in subplot c) to f). The measurement results fit well to the simulation while the sensitivity for larger angels tends to be lower than expected. The largest deviation is visible in subplot a) and cannot be explained with the applied model.

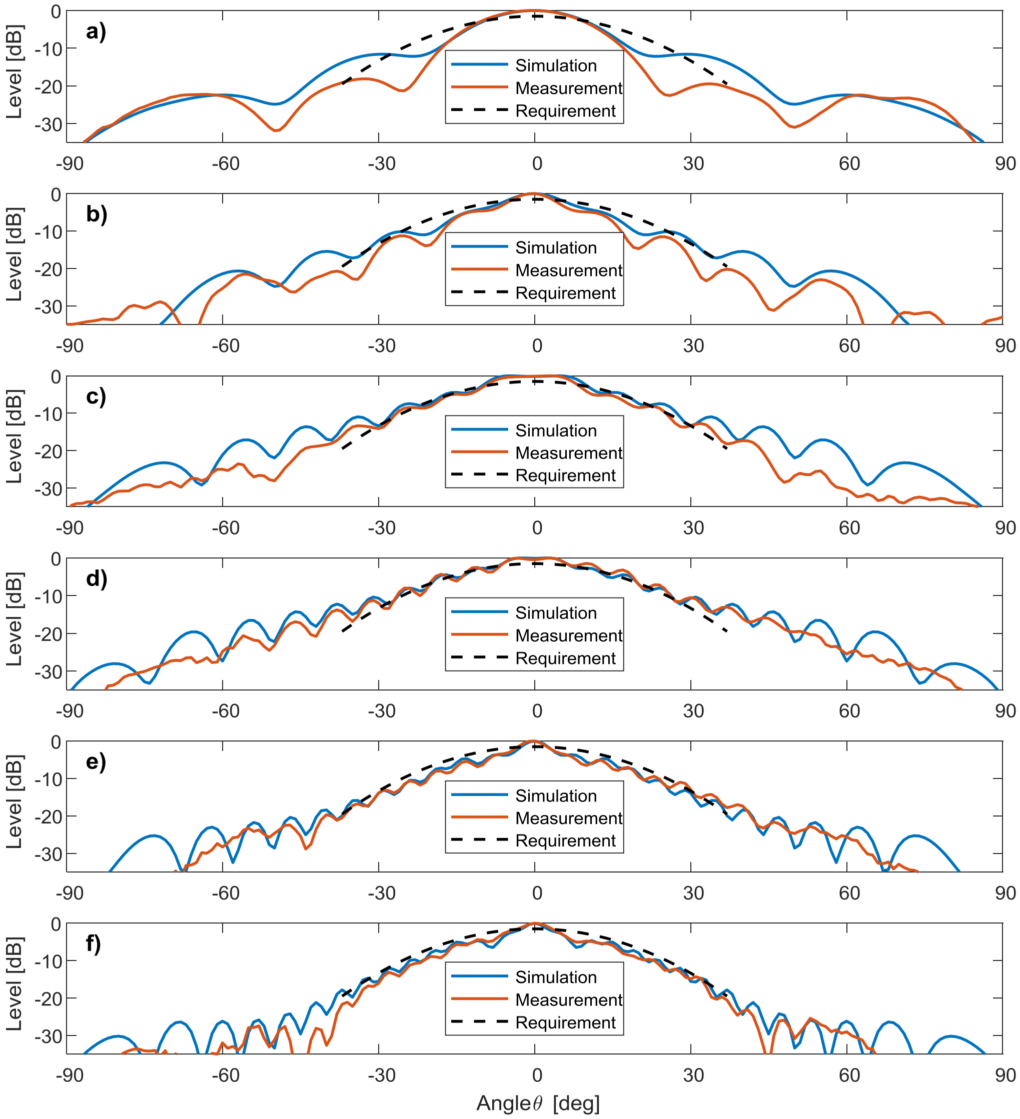

*Fig.2: Comparison between Calculated and Measured Beampattern of ‚Contour 1' for frequencies* $f = f_0, 2f_0, ..., 6f_0$ *with the lowest in plot a) to the highest in plot f)*

For ‚Contour 2„ in Fig. 3 the approximation of the flat top desired pattern is least optimal in subplot b) which results from the low effect of the curvature while in subplot a) the obtained main lobe already provides the desired behaviour. The approximation achieves excellent results for the frequencies in subplot c) to f). The measurement results also fit well to the simulation with comparable results to the ones achieved with ‚Contour 1„ besides that the deviation from the model in subplot a) insures as desired by the optimization goal an even lower sensitivity outside of the operating sector.

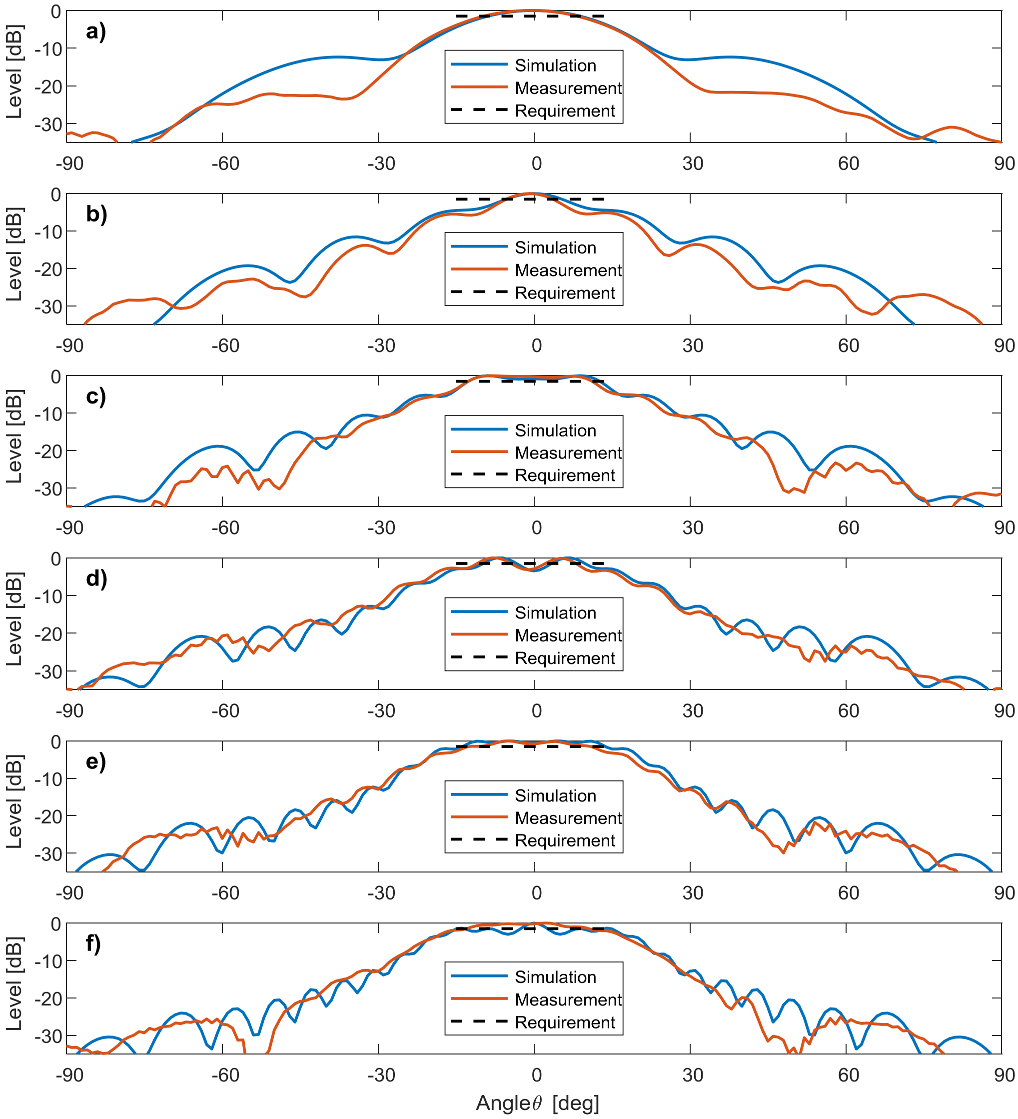

*Fig.3: Comparison between Calculated and Measured Beampattern of ‚Contour 2' for frequencies* $f = f_0, 2f_0, \ldots, 6f_0$ *with the lowest in plot a) to the highest in plot f)*

## 6. CONCLUSION

In this paper we showed results for the optimization of a conformal contour of a compact ceramic to achieve a desired transmission characteristic over a large bandwitdh. The applied discrete source model was enhanced by a sophisticated element characteristic to obtain an overall realistic result. With the compact size of the transducer the approximation of the desired sensitivity characteristic was less optimal for lower frequencies but showed a good approximation from medium to high frequencies for the two investigated cases. The modeled behavior matched well with conducted measurements where only the low frequency behavior showed larger deviations. Hence the proposed modeling can also applied for differing required characteristics. Further improvements could be achieved by further exploiting exemplary FEM simulations for more realistic element characteristics.

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