ENHANCED SONAR IMAGE RESOLUTION USING COMPRESSIVE SENSING MODELLING

Andreas Gällström[a,b] , Louise Rixon Fuchs[a,c] and Christer Larsson[a,b]

[a] Saab AB, SE-581 88 Linköping, Sweden
[b] Department of Electrical and Information Technology, Lund University, P.O. Box 118, SE-221 00 Lund, Sweden
[c] KTH Royal Institute of Technology, School of Electrical Engineering and Computer Science, Teknikringen 14, SE-100 44 Stockholm, Sweden

Contact author: Andreas Gällström, Saab AB, SE-581 88 Linköping, Sweden.
Email: andreas.gallstrom@saabgroup.com

Abstract: *The sonar image resolution is classically limited by the sonar array dimensions. There are several techniques to enhance the resolution; most common is the synthetic aperture sonar (SAS) technique where several pings are added coherently to achieve a longer array and thereby higher cross range resolution. This leads to high requirements on navigation accuracy, but the different autofocus techniques in general also require collecting overlapping data. This limits the acquisition speed when covering a specific area.*
We investigate the possibility to enhance the resolution in images processed from one ping measurement in this paper using compressive sensing methods.
A model consisting of isotropic point scatterers is used for the imaged target. The point scatterer amplitudes are frequency and angle independent. We assume only direct paths between the scatterers and the transmitter/receiver in the inverse problem formulation. The solution to this system of equations turns out to be naturally sparse, i.e., relatively few scatterers are required to describe the measured signal.
The sparsity means that L1 optimization and methods from compressive sensing (CS) can be used to solve the inverse problem efficiently. We use the basis pursuit denoise algorithm (BPDN) as implemented in the SPGL1 package to solve the optimization problem.
We present results based on CS on measurements collected at Saab. The measurements are collected using the experimental platform Sapphires in freshwater Lake Vättern. Images processed using classical back projection algorithms are compared to sonar images with enhanced resolution using CS, with a 10 times improvement in cross range resolution.

Keywords: *Sonar, imaging, resolution, Compressive Sensing, Synthetic Aperture Sonar*

1. INTRODUCTION

The enhancement of the spatial resolution in sonar imaging is a classical research field in array signal processing with several different approaches, one example being Synthetic Aperture Sonar (SAS) systems, where data from a number of subsequent measurements are combined through signal processing, resulting in an increased along-track resolution in SAS-imaging. The imaging is often performed using the back-projection algorithm in practice. The back-projection algorithm is a fast and robust method to solve the inverse problem, which gives reliable results. The drawback with back-projection is that it suffers from resolution and ambiguity limitations related to the frequency bandwidth, aperture size and sampling step sizes.

Methods that can extract more information from the available data have been developed in Compressive Sensing (CS). These methods are based on minimizing the l_1-norm of the solution and require that the solution of the inverse problem, in this case the SAS-image, is relatively sparse.

The paper is organized in the following way: Chapter 2 is a brief introduction to CS and the model used in this work. In chapter 3, two examples showing the utilization of this framework is shown – demonstrating possibilities of these techniques for sonar data.

2. COMPRESSIVE SENSING

Non-stringently expressed, the Nyquist-Shannon sampling theorem states that the sampling rate of a time-continuous signal has to be twice its highest frequency in order to ensure reconstruction. Therefore, it comes as a surprise that, under certain assumptions, it is possible to reconstruct signals when the number of available measurements is smaller than expected based on the Nyquist-Shannon theorem. The underlying assumptions for this is based on that the signal is sparse in some domain, in this case the sonar image.

A signal is called sparse if most of its components are zero. Another perspective on this is that many signals are compressible, i.e. they can be well approximated by sparse signals. This explains why the family of different compression techniques (such as JPEG, MPEG, or MP3) work so well.

The rise of interest in leveraging CS for signal processing applications has several reasons: a combination of development of theory, faster available algorithms, and faster computers. The field was pioneered by Candés et al in a publication 2004[1]. One early publication reporting an application in magnetic resonance imaging can be found in [2].

2.1 Problem Formulation

CS can be applied to several sonar frameworks. In this work, a transmitter sends out a properly designed acoustic signal, the sonar pulse, which is scattered from objects, for example on the sea floor. An array of receivers then acquires the acoustic signal resulting from the scattered waves. This can be modelled as an inverse problem:

$$Ax = y, \tag{1}$$

where the forward operator $A \in \mathbb{C}^{mxN}$, vector $x \in \mathbb{C}^{N}$ is the process variable, and measured sonar signal $y \in \mathbb{C}^{m}$. N and m are the dimensions of the operators. Normally, this problem is underdetermined (m<<N).

The problem to find the sparsest solution is formulated as an optimization based on the l_0 norm. This is, in general an NP-hard problem, therefore the optimization problem is relaxed to the l_1-norm:

$$\min\|x\|_1 \ subj. \ to \ \|Ax - y\|_2 \leq \sigma \tag{2}$$

Where the indices 1 and 2 denote the l_1 and l_2-norms, respectively, and σ is an estimate of the noise.
In this work, the quadratic constrained l_1-minimization problem has been used, in the SPGL1 implementation ([3]). The relaxation step is defined according to [1].
The model is based on isotropic and frequency independent point scatterers. Equation (3) shows the general expression for the forward-propagator acting on x, also taking into account the travel distance $|r - r'|$, frequency ω and at time t(see for example reference [5] for more information).

$$Ax = \frac{x\left(t - \frac{|r - r'|}{c}\right)}{|r - r'|^2} e^{i\omega t\left(t - \frac{|r - r'|}{c}\right)} \tag{3}$$

In this formalism, the inverse forward-propagator (i.e. back-propagator) acting on y is defined as:

$$\hat{A}^{-1}y = \sum_{n=1}^{m} w_n y_n(t) \tag{4}$$

The back-propagator is the classical delay-and-sum (see for example reference [4]), where $y_n(t)$ is the measured sonar signal from element n at time t, w_n is an appropriate window function coefficient. Both the forward-propagator and the back-propagator are used in the minimization algorithm described above.
The output from the minimization algorithm is the vector $x \in \mathbb{C}^N$, describing the sonar image. The results can be visualized using the forward projector for a specific array setup (i.e. the same geometry and configuration as used for the collecting the sonar signal) or another – synthetic-array with different geometry, for example longer array with more elements to achieve higher resolution.

3. RESULTS AND DISCUSSION

The measurements are collected using the experimental platform Sapphires in the freshwater Lake Vättern. Sapphires has a side-looking sonar array giving a SAS resolution of <4x4 cm, using conventional back-projection, combining data from several pings using auto-focussing. Results using the method described above are presented, both from one ping data and from a combination of several pings. To simplify comparisons, all measurements are covering the same objects, namely a rope loop. The images show $\|C\|_2^{1/3}$, to decrease the dynamics in the results (C being the complex valued back-projected sonar image).

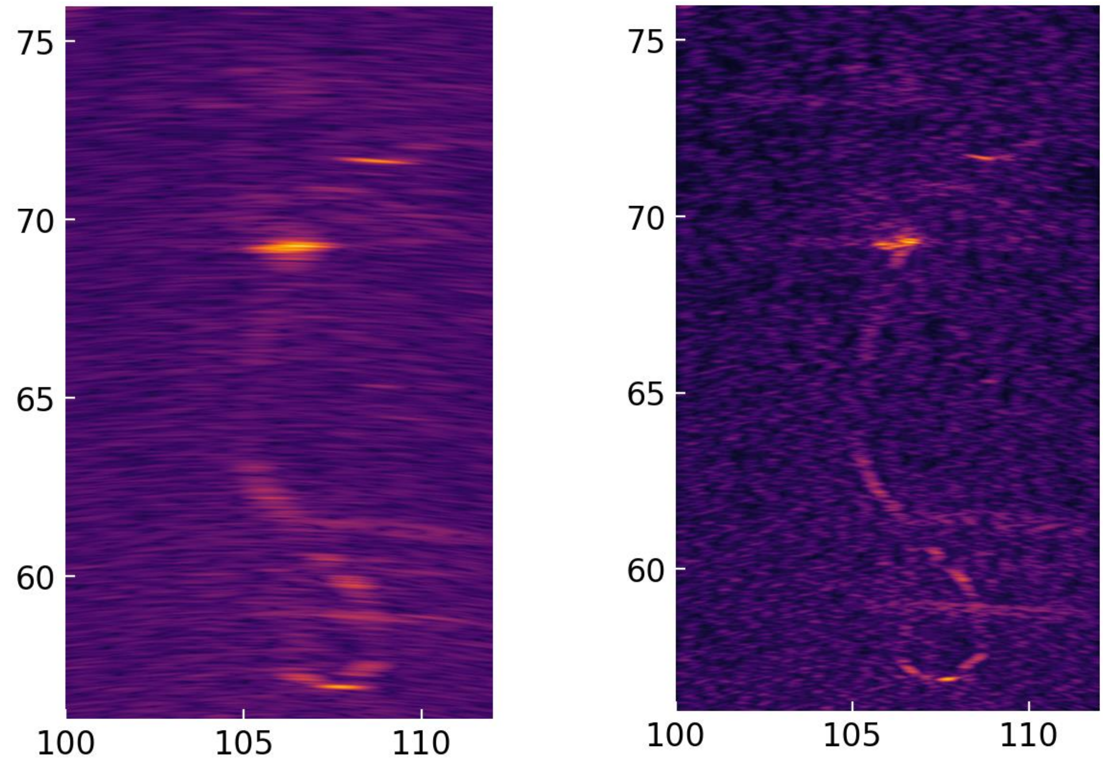

Fig. 1, Enhanced resolution from one ping data. Left image showing delay-and-sum on raw measurement data, right the enhanced image based on CS-results.

An example result is visualized in figure 1, where the image to the left shows the image using the delay-and-sum formulation on the measurement data. The image to the right shows the CS result. The CS results have been achieved using the results from the minimization together with the forward- and backwards-propagator for a synthetically extended array (but keeping the inter-distance between the receivers). In this case, the synthetic aperture is 16 times longer than the measurement aperture.

The resolution is significantly enhanced by using the model described above together with the l_1-minimization.

Worth mentioning is that the sparsity is generally in this study is around 10%, i.e. 90% of the vector x is zero-valued.

In order to compare with the more traditional synthetic aperture sonar images, different measurements covering the same area are used. In this case, the measurement apertures are not overlapping; the apertures do not partially overlap but are separated with several meters. This prevents the use of most of the traditional micro-navigation/autofocus techniques. Figure 2 shows two results; the left image is constructed using traditional techniques but incoherently added. The image to the right shows the results using CS based on the same measurement data as the left images, and here each measurement is used as input for optimization algorithm and thereby synthetic sonar data is achieved the same way as described above. The results from the three measurements are also incoherently added in this image.

We have demonstrated that CS is a promising tool for improving the spatial resolution in sonar imaging, for both one-measurement techniques as well as multiple measurements techniques (such as SAS). For alternative methods, see for example [6] using Deep Learning based methods.

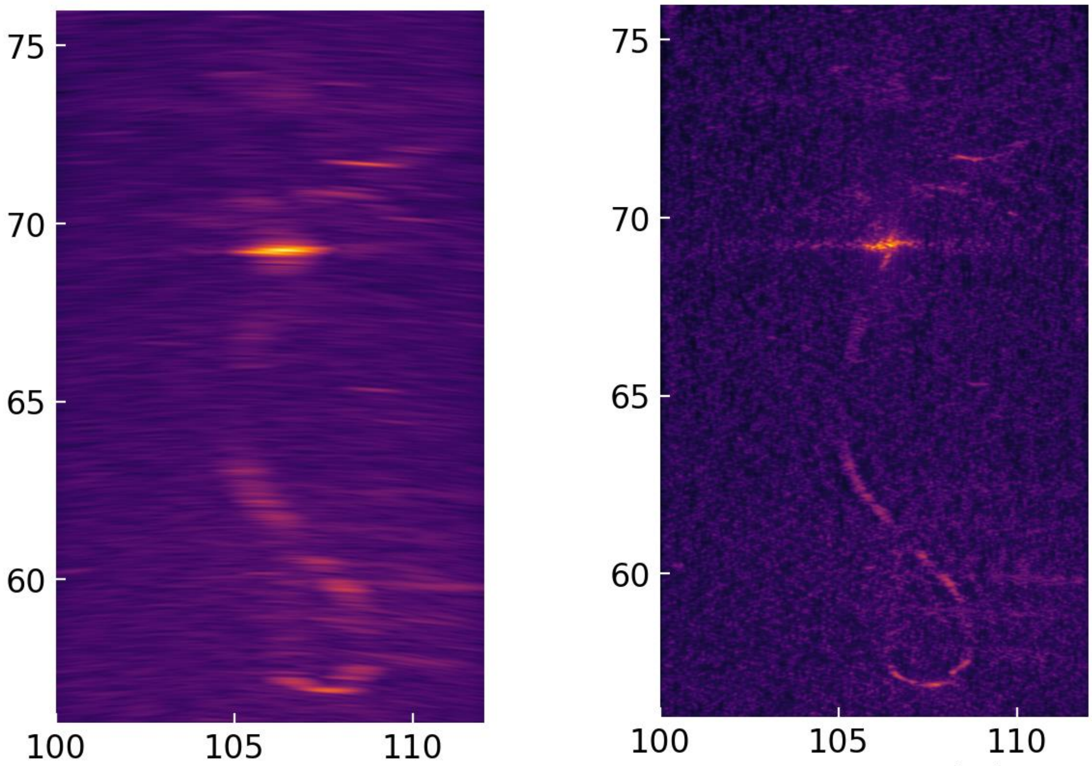

Fig. 2, *Enhanced resolution from three data measurements. Left image showing delay-and-sum on raw measurement data, right the enhanced image based on CS-results.*

ACKNOWLEDGEMENTS

This work was supported by the Swedish Foundation for Strategic Research through Strategic Mobility grants.

REFERENCES

[1] **E.J. Candés, J. Romberg, T. Tau**, *IEEE Trans. Inform. Theory*, 52 489-509 (2004)
[2] **M. Lustig, D. Donoho, J. M. Pauly,** *Magn. Reson. Med.*, 58 1182-95 (2007)
[3] **E. van den Berg, M. P. Friedlander**, *SIAM J. on Scientific Computing*, 31(2): 890-912 (2008)
[4] **R. E. Hansen**, Introduction to Synthetic Aperture Sonar, Sonar System, Prof. Nikolai Kolev (ed.) (2011), ISBN: 978-953-307-345-3
[5] **R. J. Urick**, principles of underwater sound, 3rd edition (1983), ISBN: 9780932146625
[6] **L. Rixon Fuchs, C. Larsson, A. Gällström**, Deep Learning based Technique for enhanced sonar imaging, In *UACE 2019, Crete-Greece*, 2019