VON MISES PRIOR FOR PHASE-NOISY DOA ESTIMATION: THE VITAMIN ALGORITHM

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Abstract: *Sound waves in the ocean are affected by the space and time variabilities of the propagation medium. These fluctuations, mainly caused by internal waves such as tides and gyres, can lead to a loss of phase information in measured wave-fronts, and make hardly predictable the true location of a source. As a consequence, the performance of classical direction-of-arrival (DOA) estimation algorithms are significantly degraded. An important literature addresses this issue by considering either the phase as non-informative or the environment as a noise with no physical information. In this work, we propose to introduce a phase prior inspired by random fluctuation theories. This prior is combined with a sparsity assumption on the number of expected DOAs and exploited within a Bayesian framework. The contributions of such an approach are two-fold: by the use of suitable prior information (small number of DOAs and phase distortion), it allows an estimation of DOAs from a single snapshot, while simultaneously providing a posterior estimation of the mean fluctuations of the propagation medium. Bayesian inference can be performed in different ways. Among the different possible procedures, we chose here to resort to a Bethe approximation and a message-passing approach recently considered in compressive sensing setups. The resulting algorithm places in the continuation of our previous works. The main improvement lies in the probabilistic model used to describe the phase distortion. Here we use a Multivariate Von Mises distribution, more suitable to directional statistics and still fitting the simplified theory of phase fluctuation. Numerical experiments with synthetic datasets show that the proposed algorithm, dubbed as VITAMIN for ``Von mIses swepT Approximate Message passINg'', presents interesting performance compared to other state-of-the-art algorithms. In particular, in the considered experiments, VITAMIN behaves well regarding its robustness to additive noise and phase fluctuations.*

Keywords: *Compressed Sensing, DOA Estimation, Message Passing, WPRM, Bayesian Estimation.*

1. INTRODUCTION

Wave propagation through random media is at the heart of many applications (RADAR, acoustics, telecommunications…). Studies in this research field aim at characterizing the fluctuations of the propagation medium and their impact on a transmitted signal to improve associated processing. In this respect, the problem of direction-of-arrival (DOA) estimation occupies an important place in the literature. However, despite the large number of methods proposed, very few explicitly address the problem of fluctuating environments.

DOA estimation in well-known environments simply corrupted by additive white Gaussian noises constitute indeed the wide majority of the approaches. In that family, we find first the very popular beamforming approach [1], which relies on a simple inversion of the measurement matrix. Other methods put additional assumptions about the nature of the source signal. For example, some 'high resolution' techniques rely on the hypothesis of orthogonality between signal and noise subspaces like in the MUSIC algorithm [2]. More recent contributions propose to exploit a sparsity assumption on the number of DOA to recover, leading to so-called 'compressive' beamforming techniques [3].

To the best of our knowledge, DOA estimation in noisy environments has been mostly addressed from two major perspectives: the corrupting effect of the propagation medium on measurement is modelled either as a non-informative phase perturbation [4] or informative but uncorrelated phase noises [5].

Our previous work [6] started from this vein of methods. We tried however to go further by considering a perturbation model compatible with some perturbation regimes observed and defined by [7, 8]. Given a high-frequency approximation, we were able to modelize the perturbation as a multiplicative phase noise following a multivariate Gaussian law with a covariance matrix carrying the information related to the strength of the fluctuations. This model was exploited through a Message Passing Algorithm, resulting in a procedure named paSAMP [9].

In this paper, we refine our noise model to be more suitable for the issue of phase perturbation, using a Multivariate Von Mise Prior, more adapted to directional statistics [10]. As a result, we propose an improved version of paSAMP: the VITAMIN (for Von mIses swepT Approximate Message passINg) algorithm.

2. CHARACTERIZATION OF RANDOM FLUCTUATION IN THE OCEAN

We use here the results of works initiated in [7] and then updated in [8] to characterize the impact of multi-scale fluctuations of the ocean on a measured signal. Given a certain fluctuation strength of the internal waves, the signal can be corrupted as a function of its frequency, the propagation range and other interactions bottom-surface. Considering a high frequency approximation, we can define three saturation regimes which can be easily described through geometrical optics:

- The unsaturated regime: variations are not strong enough to alter the source ray. In this cas, there is no perturbation on the measured DOA.
- The partially saturated regime: stronger variations cause a diffraction phenomenon of the source ray. As a result, the sensors measure a coherent ray and multi-rays due to this spread. This spread is directly linked to the strength of the fluctuations and impacts the consistency of the wave propagated in the random medium. Beyond a so-called 'coherence-length', we observe a loss of consistency, this length can be retrieved from measurement and easily modelled as a covariance structure within a random phase noise.

- The fully saturated regime: this regime can be observed when the variations are too strong, the micro-rays spread beyond the coherence ray. In that case, we observe a total loss of information on the phase noise statistics.

In this paper, we focus on DOA estimation in presence of a partially saturated regime.

3. FRAMEWORK

Observation model

Considering a *N* sensors array, one snapshot of the acoustic signal received on the *n-th* sensor can be written as:

$$y_n = e^{j\theta_n} \sum_{m=1}^{M} d_{nm} x_m + \eta_n \tag{1}$$

with θ_n the phase noise due to the spatial fluctuations of the medium, d_{nm} an element of the plane wave dictionary D with $d_m = [e^{j\frac{2\pi}{\lambda}\Delta \sin(\phi_m)} \dots e^{j\frac{2\pi}{\lambda}\Delta N \sin(\phi_m)}]^T$, Δ the sensor spacing and λ the signal wavelength. The aim here is to retrieve $x = [x_1, \dots, x_M]^T$ whose components index the atoms of D (i.e. the directions of arrival ϕ_m) in presence of phase noise and additive noise η_n.

Bayesian framework : prior assumptions

Prior on the sources

As a first assumption, we consider a small number of DOA: **x** is assumed to be sparse. Within a Bayesian framework, this constraint can be formulated as a Bernoulli-Gaussian prior on **x** [11], so that, for each x_m :

$$p(x_m) = \rho \mathscr{CN}(x_m; m_x, \sigma_x^2) + (1 - \rho)\delta_0(x_m) \tag{2}$$

where ρ is the Bernoulli parameter, standing for the 'sparse rate' , i.e. the probability for x_m to be non-zero, $\mathscr{CN}(x_m; m_x, \sigma_x^2)$ stands for the circular Gaussian distribution with mean m_x and variance σ_x^2. Finally, δ_0 is the Dirac distribution.

Prior on the phase noise

As mentioned above, we choose the prior on θ_n with regard to previous works dealing with phase retrieval algorithms used for solving DOA estimation problem, studies [7,8] addressing the statistical impacts of the fluctuations on the measured signal and works about directional statistics [12,10]. In this respect, the multivariate Von Mises distribution appeared to be particularly suitable [10]:

$$p(\boldsymbol{\theta}) \propto \exp(\boldsymbol{\kappa}^T \mathbf{c}(\boldsymbol{\theta}, \boldsymbol{\mu}) - \mathbf{s}(\boldsymbol{\theta}, \boldsymbol{\mu})^T \Lambda \mathbf{s}(\boldsymbol{\theta}, \boldsymbol{\mu}) - \mathbf{c}(\boldsymbol{\theta}, \boldsymbol{\mu})^T \Lambda \mathbf{c}(\boldsymbol{\theta}, \boldsymbol{\mu})) \tag{3}$$

with parameters $\boldsymbol{\mu} = [\mu_1, \dots, \mu_M]^T, \boldsymbol{\kappa} = [\kappa_1, \dots, \kappa_M]^T$and Λ relative to a precision matrix, also introducing the cosine and sine vector relative to the angular noise,
$\mathbf{c}(\boldsymbol{\theta}, \boldsymbol{\mu}) \triangleq [\cos(\theta_1 - \mu_1), \dots, \cos(\theta_M - \mu_M)]^T, \mathbf{s}(\boldsymbol{\theta}, \boldsymbol{\mu}) \triangleq [\sin(\theta_1 - \mu_1), \dots, \sin(\theta_M - \mu_M)]^T$.

Prior over the additive noise

Classically, we finally consider the additive noise η_n as a white gaussian noise, with zero mean and a variance σ^2.

4. BAYESIAN ESTIMATION OF THE DOA

Considering model (1)-(3), we can formulate the DOA estimation problem as a Minimum Mean Square Error problem:

$$\hat{\mathbf{x}} = \underset{\tilde{\mathbf{x}}}{\operatorname{argmin}} \int_{\mathbf{x}} ||\mathbf{x} - \tilde{\mathbf{x}}||_2^2 \, p(\mathbf{x}|\mathbf{y}) d\mathbf{x} \tag{4}$$

where $p(\mathbf{x}|\mathbf{y}) = \int_{\theta} p(\mathbf{x}, \boldsymbol{\theta}|\mathbf{y}) d\boldsymbol{\theta}$ is the posterior distribution over $\mathbf{x}$ marginalized over the phase perturbation $\boldsymbol{\theta}$. Because of this marginalization, solving (4) leads to an intractable problem. One possible way to circumvent the issue is to search for an approximation of $p(\mathbf{x}, \boldsymbol{\theta}|\mathbf{y})$. More specifically, we propose here to resort to variational Bayesian approximations which try to approach $p(\mathbf{x}, \boldsymbol{\theta}|\mathbf{y})$ with some suitable factorisations minimizing the Kullback-Leibler divergence (i.e. the loss of information between the true distribution and the factorized one). Among them, we focus on the so-called Bethe approximation.

From Bethe approximation to the paSAMP and VITAMIN algorithms

The Bethe approximation relies on a factorisation of the posterior distribution of interest according to some clusters of variables. Minimizing the Kullback-Leibler divergence under such a penalization can then be well-handled by Message Passing algorithms [13], namely methods based on Belief Propagation. Among them, the Swept Approximate Message Passing algorithm [14] was proposed to solve non-linear but still component-wise problem (namely, measurements (1) are assumed to be independent from each other). In [6], we extended this approach to a Gaussian phase noise, leading to the paSAMP algorithm. Here, we adapt it to the multivariate Von Mises distribution (2). Due to space limitations, we omit the derivations of the algorithm. We refer however the reader to [15] for more details. We named the approach VITAMIN (for Von mIses swepT Approximate Message passINg) algorithm.

5. PROOF OF CONCEPT

In this section we discuss the performance of the VITAMIN algorithm to justify the interest in refining the phase noise model with more 'directional-inspired' methods.

For this, we run this algorithm on a set of simulated data according to a specific configuration. We want to recover the directions of arrival of k plane waves over a set of $N = 32$ potential angles with an antenna of $M = 32$ sensors. We consider here a simple setup where the phase noise obey a univariate Von Mises law. This corresponds to model (3) with parameter $\Lambda = 0_{32\times32}$. For the discussed result, we consider homogenous $\boldsymbol{\kappa}$ and set $\kappa_n = 4$ for all n, this value is equivalent to a Gaussian noise with a standard deviation of 0.25 radians, corresponding to an important angular perturbation.

We set the parameter $\lambda/\Delta = 4$ and for each of the k incident waves, each coefficient of $\mathbf{x}$ is designed as a realisation of distribution (2) with $m_x = 0 + j \times 0$, $\rho = K/M$ and $\sigma_x^2 = 0.1$.

We want to observe the contribution in terms of reconstruction performance of such a prior. To do this, we observe the normalized correlation $\frac{|\mathbf{x}^H \hat{\mathbf{x}}|}{\|\mathbf{x}\|\|\hat{\mathbf{x}}\|}$ between the true vector $\mathbf{x}$ and its estimate $\hat{\mathbf{x}}$ as a function of the additive noise variance and for different numbers of sources k. To obtain stable results we mean this performance index over 200 trials, corresponding to the number of realizations of $\mathbf{x}$ and angular noise.

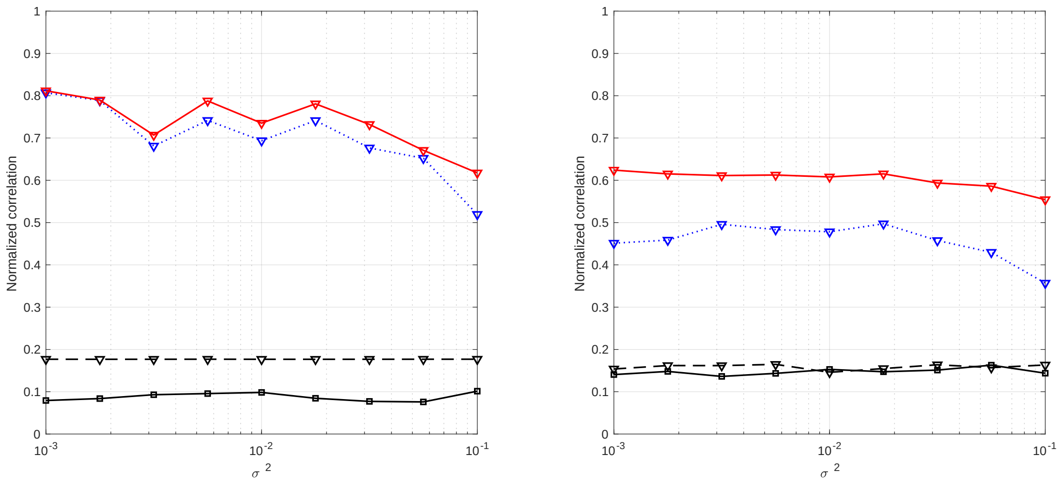

Figure 1: Normalized correlation regarding the additive noise variance σ^2. Here we observe the performance of conventional beamforming (squared line), prSAMP(dashed line), paSAMP(dotted line) and VITAMIN (triangle, solid line), one source (left) and ten sources (right).

Figure 1 illustrates the performance of VITAMIN and - even if the equivalence between the Von Mises distribution and the Gaussian distribution can be made for small variances - the importance of considering a directional prior in presence of phase noise. Indeed, we can see on this figure that VITAMIN outperforms the compressive beamforming and the prSAMP algorithm which do not integrate any informative prior over the phase noise. Moreover, the performance gap between VITAMIN and paSAMP (which exploits a Gaussian prior over the phase noise) tends to increase with the number of sources. This may due to the loss of information while considering an approximated prior: with a small number of sources, the loss is minimized but it tends to rise with the amount of information.

CONCLUSION

In this work, we proposed a new implementation of prSAMP with a modelization of phase noise from directional statistics. Such an approach can be justified by the fact that in such compressive sensing method, good prior knowledge is necessary to achieve perfect estimation of $\mathbf{x}$. After derivation of the calculations, we are able to propose the VITAMIN algorithm, allowing us to perform a better DOA estimation than algorithms which integrate a non-informative or Gaussian noise prior.

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