EFFICIENT MODELING OF 3D ACOUSTIC PATHS TO IMPROVE SOURCE TRIANGULATION

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Abstract: *Hydroacoustic signals recorded on the International Monitoring System (IMS) network have proven their capability to identify the origin of intense sound sources. A recent example is the detections CTBTO used to identify the search area where the lost Argentine submarine ARA San Juan was found. To triangulate a source, isochrones or surfaces of equal travel time are propagated from each station using a global model of ocean sound speed. With detections on two stations, the intersection of these isochrones form a line, which reduces to point when time of occurrence is known. With detections on 3 stations, isochrones intersect at the source origin and establish a time of occurrence, noting that confidence is limited by accuracy of the propagation model. Finding a detection on three stations however is not always possible, and as such azimuthal information garnered from coherent processing of the hydrophone triads that make up each IMS station provide another means to bound the triangulation. Furthermore, when additional arrivals distributed in azimuth can be associated with a particular source, e.g. delayed arrivals having a common spectral feature, these 3-dimensional paths can be modelled to establish additional isochrones. This serves as an addition of "virtual" stations to improve triangulation, and is demonstrated using the bathymetrically refracted 3D features observed in the signal of the ARA San Juan. Spectral and cepstral features allow for unambiguous association of the 3D arrivals recorded HA10-Ascension and HA04-Crozet. This study concludes with an assessment of the accuracy, limitations and efficiency of 3D modelling techniques based on hybrid of rays, acoustic modes and the parabolic equation.*

Keywords: *3D acoustics, normal modes, long-range propagation*

1. INTRODUCTION

On November 15th, 2017 an intense sonic impulse was recorded by two Comprehensive Nuclear Test Ban Treaty (CTBTO) hydroacoustic stations. This sound was generated by the catastrophic failure of *ARA San Juan* (a submarine) hull as it sank to the bottom of the ocean.[1] This intense signal propagated 6000 km to Ascension Island (HA10) and 8000 km to the Crozet Islands (HA04). The signal contains a wealth of information relating to the origin of the sound as well as oceanography.

Echoes of intense impulses produce a long duration coda, which in the case of a nuclear detonation lasts for hours.[2] The coda of the *San Juan* impulse observed at the Mid-Atlantic station was detectable for over 15 minutes, and consisted of delayed arrivals from multiple directions.[3] These 3D arrivals were sound energy redirected, i.e. refracted, by ocean topography (bathymetry).[4] This focused study develops an acoustic propagation model to reliably reproduce three-dimensional (3D) features (an example is the dotted line in Fig. 1).

This paper is organized as follows: first, a forward model reproduces characteristics of the observed signal dispersion. This forward model provides important information relating to modal arrival times needed for triangulation. Next, timing of distinct modal arrivals is modelled with adiabatic mode theory, and a piecewise approach efficiently reconstructs the energy path of 3D arrivals. The paper concludes with triangulating *San Juan* based on three arrivals (those shown in Fig. 1).

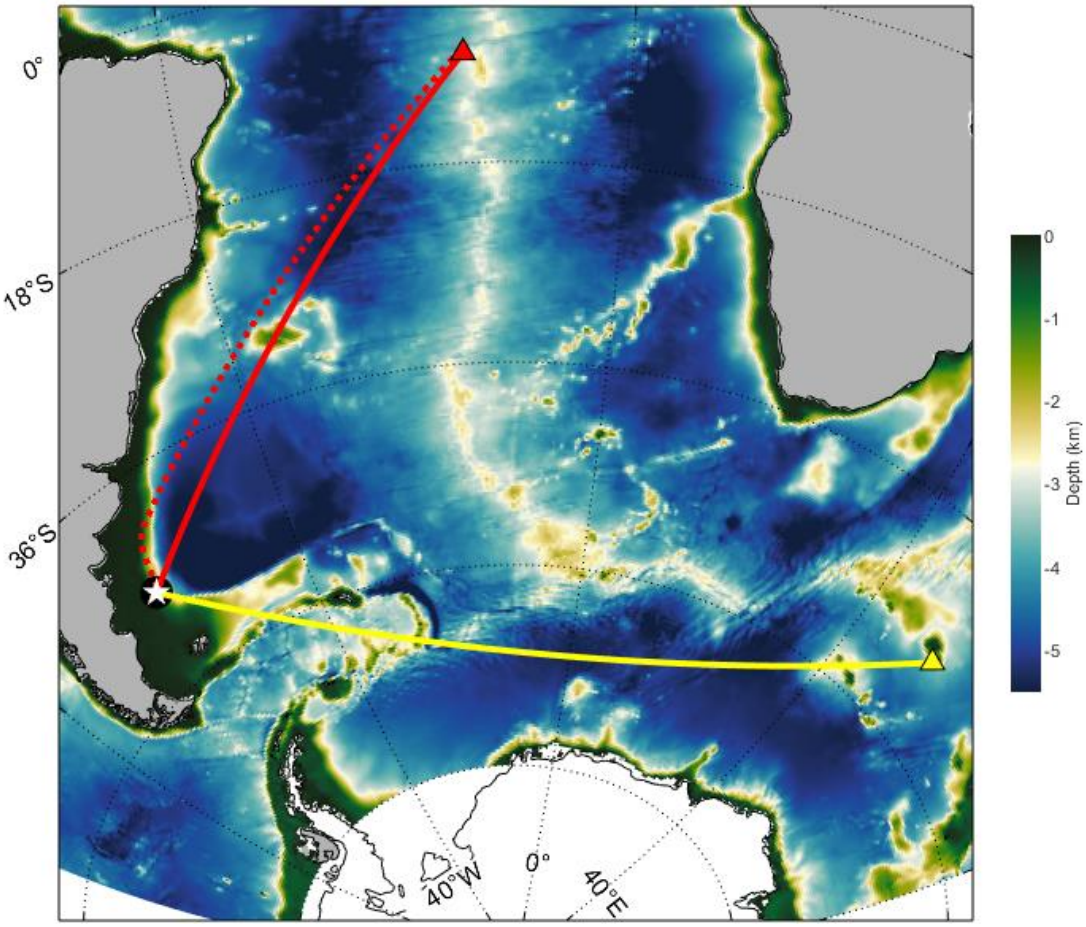

Fig. 1: Acoustics paths to H10N (red) and H04S (yellow) used for triangulation.

2. FORWARD MODELING PROPAGATION (FEATURE IDENTIFICATION)

Modelling propagation paths at megameter scale requires some careful considerations. Boundary interactions and interplay with oceanography are fundamental aspects of long-range sound propagation. This vertically confinement, or trapping of sound energy makes long range acoustic detection possible. Sound trapping by boundaries, the characteristic defining shallow water propagation, is efficient when wavefronts of the incident field propagate at a low (horizontal) grazing angle. For bottom reflections, sound is nearly perfectly reflected at grazing angles less than the critical angle, $\Theta_c = \cos^{-1}(n)$ where n is the ratio of sound speed (index of refraction) across the interface. Propagation can be approximated by a set of discrete trapped modes populating propagation angles from horizontal to Θ_c. As frequency increases so does the number of modes, and the mode-1 propagation angle migrates closer to horizontal.

In the deep ocean, very little sound energy propagates to the sea floor due to a gradient in sound speed caused by increasing pressure (roughly an increase of 10 m/s per km depth). This turning of energy is more efficient than a boundary reflection, as scattering from surface roughness is avoided. In addition to upward refraction, downward refraction by temperature gradients in the upper ocean form a true sound speed duct known as the Sound Fixing and Ranging (SOFAR) channel.[3] In avoiding scattering from the sea surface and sea floor, sound in the SOFAR channel propagates great distances.

The most basic model of a propagation path in latitude and longitude is the shortest path on the surface of the Earth. These geodesics are defined here on the World Geodetic System (WGS84). Although providing a reasonable representation of the sound energy path, prediction of the arrival time also depends on the meandering depth of the sound energy. This aspect elongates the propagation path, but as speed of sound also changes with depth, dispersion in mode arrivals is geospatially dependent. The depth dependent speed of sound (sound speed profile) along the geodesic from [45.9402° S, 59.8101° W] to the two CTBTO hydroacoustic stations are shown in Fig 2; observe the SOFAR channel at HA10 (top).

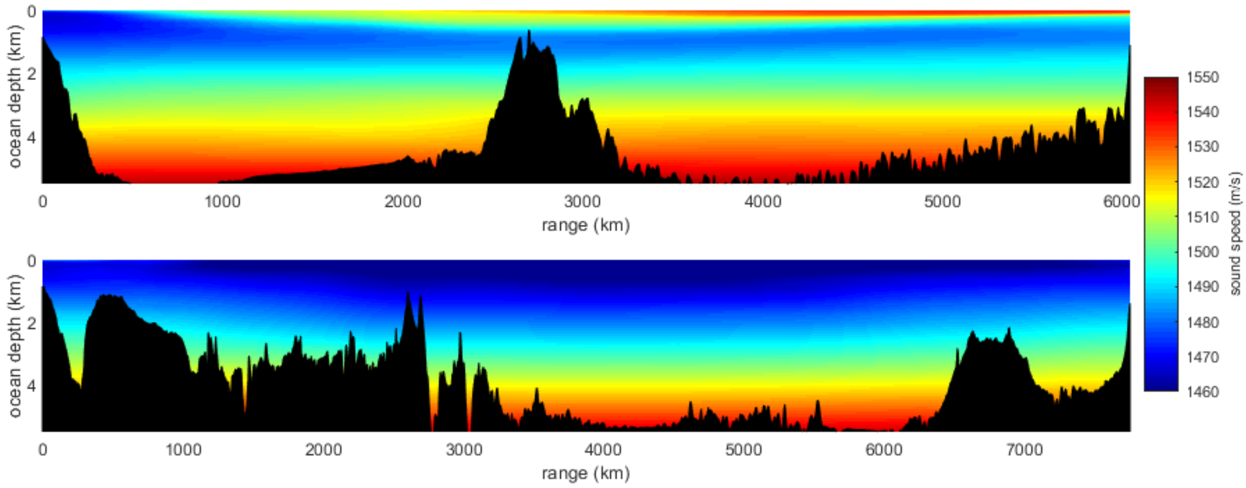

Fig. 2: Sound speed profiles along the propagation path to HA10 (top) and HA04 (bottom). Data tabulated from the World Ocean Atlas.

A two dimensional propagation model (along a geodesic and depth) provides valuable insight into the time frequency characteristics of the direct energy path. Propagation models cast in a Cartesian coordinate system must implement an Earth flattening transformation to account for path shortening in depth for a given arc length. This applies to both sound speed and ocean depth, which are multiplied by the ratio R / (R – D), where R is the radius of Earth, and D is depth.

Figure 3 shows a model of the direct field at HA10 and HA04, based on the numerical PE solver RAM-PE.[5] The time series are a Fourier synthesis of 601 discrete frequencies, with time-frequency characteristics showing a dispersed signal similar to those observed in the data (see Fig. 4). A matched field comparison can provide a precise estimate of the source range (and time); however it is worth noting the computation time to produce Fig. 3 was nearly 24-hrs. Although at long ranges such numerical methods are slow, the full wave-representation maintains a high degree of accuracy.

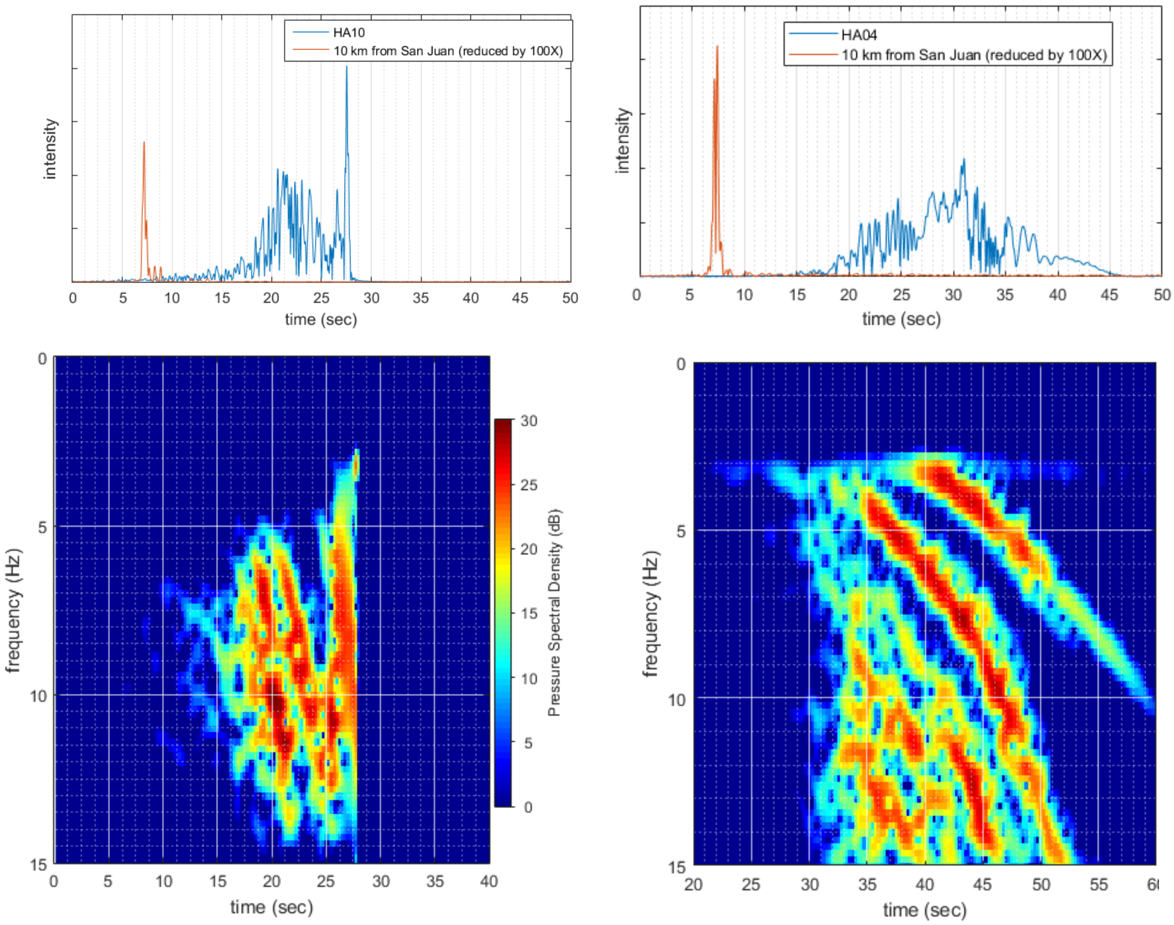

Fig. 3: [*left*] *Synthesized impulse pressure as received 10 km from the San Juan location and HA10, and [right] at 10 km and HA04. Vertical axis is intensity on a linear scale, and level at 10 km is divided by 100. Lower time-frequency plots are the corresponding he pressure spectral density at HA10 and HA04 (decibel units).*

An alternative and computationally fast model of propagation is based on adiabatic mode theory (adiabatic implies no energy is exchanged between modes). The dispersion characteristics of the *San Juan* impulse at the CTBTO stations show distinct mode arrivals. A difference in dispersion at the two stations is striking, and it is due to the different oceanography along each path (discussed in the next paragraph). Correct identification of modal arrival times is essential for triangulation through back-propagating of detected arrivals. Arrival times depend on group velocity, and an estimate of arrival time (T_n) for the n^{th} mode is given by,

$$T_n = L\frac{d\bar{k}_n}{d\omega} \quad (1)$$

where L is the distance along the geodesic connecting source and receiver, ω is frequency times 2π, k_n is the horizontal wavenumber of the n^{th} mode and the overbar indicates an average value over the propagation path.

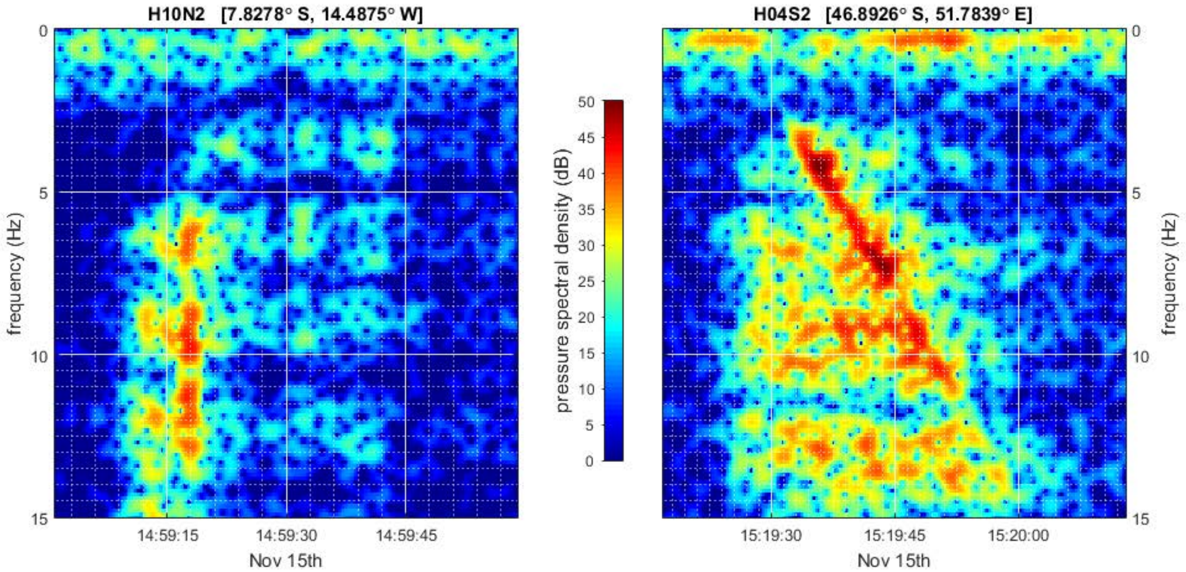

Fig. 4: Time-frequency plot of recorded direct arrival at H10N2 (left) and H04S2 (right)

Propagation to HA10 is primarily within the SOFAR channel, and mode-1 group velocity is effectively frequency independent from 5-15 Hz. Propagation to HA04 is within a surface duct, i.e. reflected from the sea surface and refracted by the deep ocean. Due to the increase in mode number with frequency, mode-1 is pushed closer to horizontal causing its group velocity to slow down linearly with increasing frequency. Group velocity ($d\omega/dk_n$), phase velocity and horizontal wavenumber can be readily calculated via a mode finding routine. With Eq. (1) and mode properties ascertained from the normal mode program KRAKEN [6], and frequency dependence of T_n reproduce the characteristic or mode-1 dispersion. At HA10, the mode-1 arrival is frequency independent, and at HA04 the mode-1 arrival exhibits a 2.5 second delay per Hz [see Fig. 4]. This also confirmed the peak arrival at 10 Hz corresponds to mode-1, and the delay of mode-1 from the onset of signal detection at both stations.

3. PIECEWISE MODELLING OF AZIMUTHALLY REFRACTED ARRIVALS

The 2D model lacks information about the signal coda after the direct arrival (about 30 seconds). The later part of the coda consists of bathymetrically refracted arrivals, and contains valuable information for triangulating sources. To elaborate on this, let us consider the first (and loudest) 3D arrival observed in the HA10 recording. This sound arrives roughly 90 seconds after the first detection, having travelled further to the continental slope where refraction occurred (see dotted red line in Fig. 1).

To model refracted arrivals, horizontal dependence of the adiabatic mode description is modified from a single radial coordinate to two horizontal coordinates (easting and northing). Depth dependence is maintained by the phase and group velocity description of the mode. What drives refraction are gradients in mode phase speed, and thus azimuthal coupling can be modelled with standard 2D ray tracing or PE methods where phase speed is the surrogate for speed of sound.

What we gain with the PE comes at great computational expense. This inherently narrowband simulation method requires a Fourier synthesis to reproduce a timeseries. Note again the long computation time to produce Fig 3, nearly 24-hr for 601 discrete frequency PE simulations. However, narrowband simulations can still be used to identify the path of sound energy. Energy flux streamlines depict the path of energy and are formed by marching in the direction intensity, and divergence of two neighbouring streamlines corresponds to intensity (less energy lost to heat or transmission into the sediment). In free-space, i.e. a uniform medium with no boundaries, streamlines and rays are identical, excepting in the nearfield of complicated sources.[7]

In deep water, low-frequency sound (10 Hz) follows geodesics. Shoaling of sound energy at the continental shelf can be modelled with ray tracing techniques. Ideally the ray connecting a source and receiver is in the direction of the detected energy. However, due to scattering from a realistic bathymetry, finding of such rays is not always possible. To overcome this, a PE solution with a beam starting field is implemented to construct refracted streamlines. The PE embodies the complete dynamics of the refraction process, and by using a beamed source the refracted field is isolated from the direct field [see Fig. 5].

Efficient modelling of 3D arrivals makes use of PE only for the portion of propagation into shallows (the water depth where refraction takes place, which also depends on frequency). The streamline description allows for initiation and then continuation of streamlines with rays (geodesics at 10 Hz) if they propagating into deeper water. Figure 5 shows the continuation of 10 Hz mode-1 with an intensity beam, representing the back propagated energy from H10N2; a clear path exists to the *San Juan* location (star). Of curiosity is a hint of an additional ‘whispering gallery’ path that follows contours of the continental slope. The region where PE computation of the refracted path is needed is roughly 100 km x100 km, which takes less than a minute to compute.

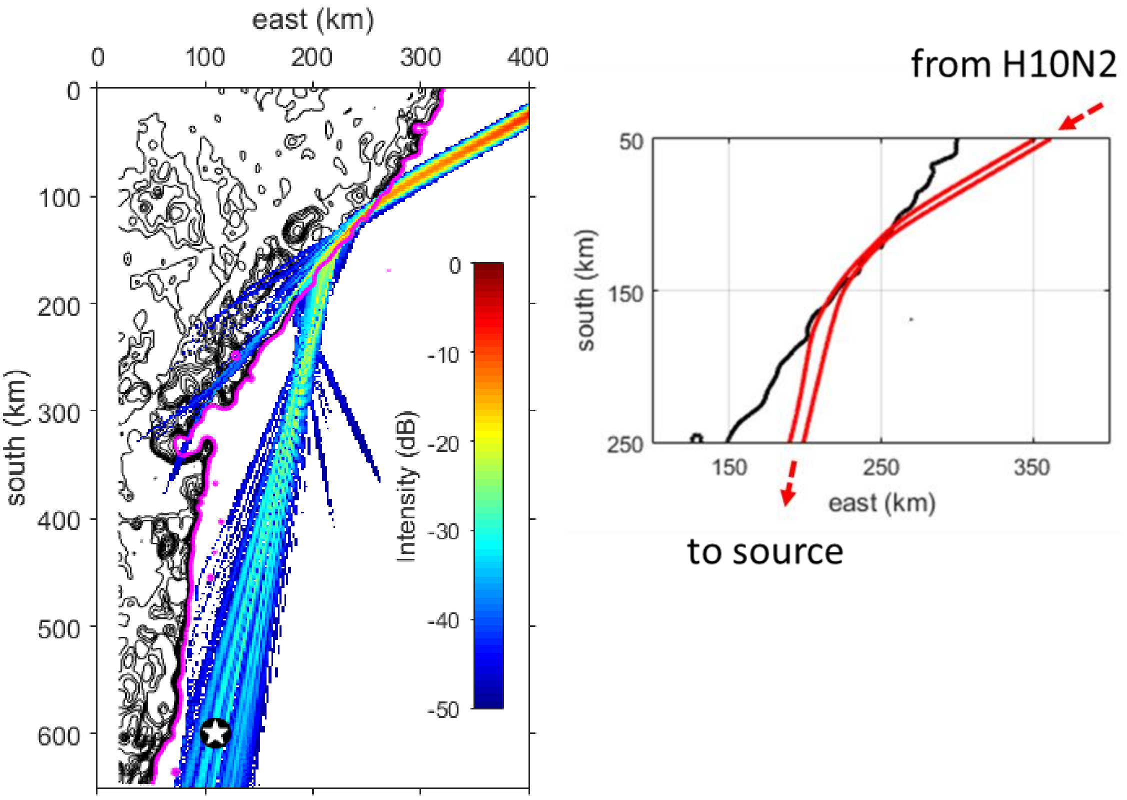

Fig. 5: [Left] Intensity beam of mode-1 at 10 Hz depicting horizontal refraction and energy path to the San Juan (star). The input beam direction is a continuation of the geodesic launched from H10N at bearing 221.3°, and the magenta line indicate the 300 m depth contour where bathymetric refraction come into play. [Right] Streamline tube built from the PE simulation, input heading 241° and output heading is 193°.

4. TRIANGULATION OF THE ARA SAN JUAN

With the 3 energy paths shown in Fig. 1 established, two paths to HA10 (Ascension) and one to HA04 (Crozet), the timing of the mode-1 arrival is calculated via Eq. 1. Since mode-1 is characterized by the peak energy arrival at 10 Hz, arrival times are extract from the data. Table 1 outlines the extracted arrival time estimates for mode-1 at 10 Hz used for triangulation. Source triangulation is at the intersection of three isochrones, or the surfaces of equal travel time that are perpendicular to propagation paths. Fig. 6 shows the intersection of the three isochrones, occuring near the location of the *San Juan* (star). With the model parameters, this estimates a source time 13:51:18 UTC.

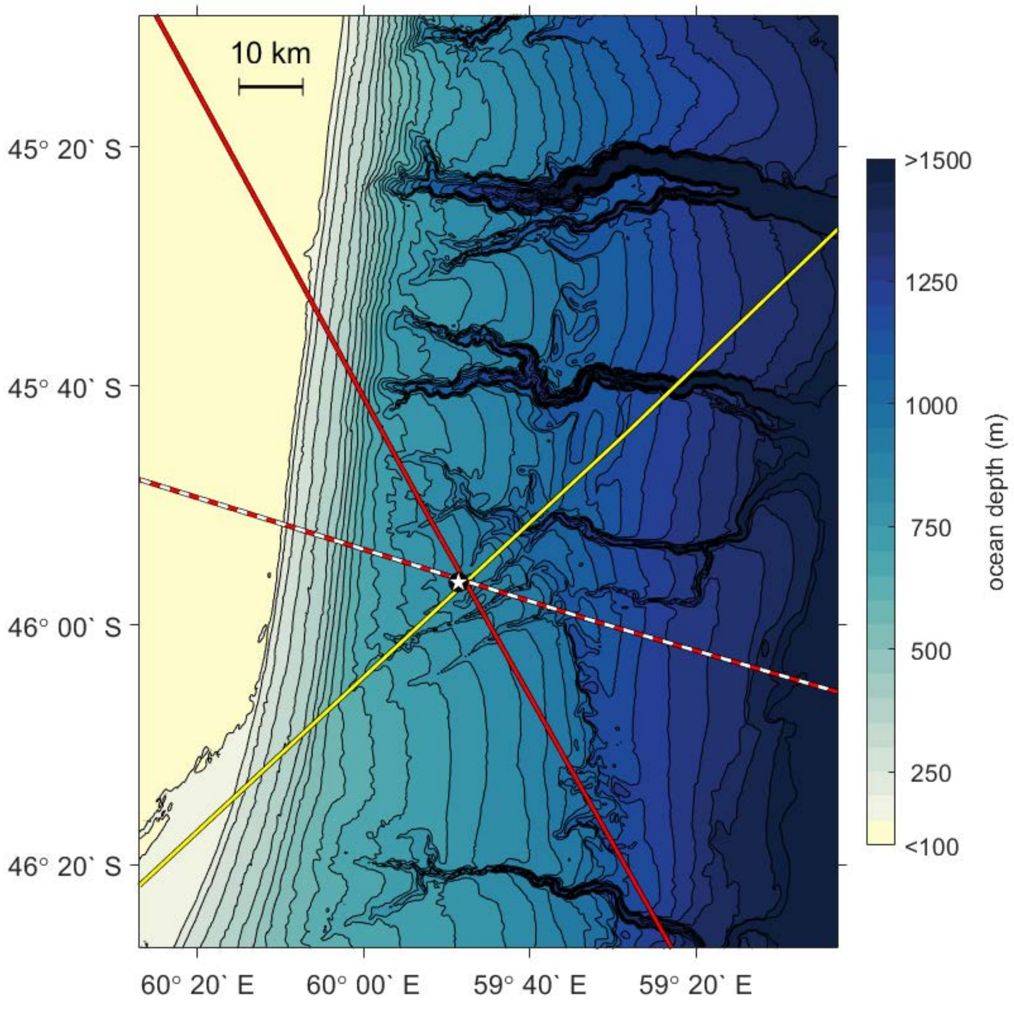

Fig. 6: Triangulation result giving a source time of 13:51:18.

MODE 1 ARRIVAL TIME	NOV 15TH	DEC 1ST
H10N2	14:59:15	21:12:06
H10S2	14:58:04	21:10:51
H04S2	15:19:44	21:33:00
REFRACTED ARRIVAL TIME		
H10N2	15:00:29	21:13:24
H10S2	14:59:23	21:12:15

Table 1: Timing of mode-1 arrivals at 10 Hz used in triangulation

5. SUMMARY AND DISCUSSION

The intense impulsive sound emitted from the submarine, *ARA San Juan*, produced a number of bathymetrically refracted arrivals. A refracted arrival observed at the Ascension Island CTBTO hydroacoustic station was modelled and used to triangulate the sound source from the detections. The energy path of this refracted arrival was reproduced through back-propagating an energy streamline by a piecewise model based on adiabatic modes. By design, azimuthal dependence of streamlines are only calculated with a computationally expensive PE where refraction occurs (at 10 Hz this is in water depth less than 300 m).

Although only one 3D path is detailed here, this technique can be applied to model additional 3D arrivals in the arrival coda and reduce uncertainty in triangulation. The beamed sources with a PE computation addresses complicated propagation the continental slope, but also applies to modelling the modulation of direct propagation around seamounts. Furthermore, this beamed energy technique can build a transfer function to account for refraction (i.e. change in propagation direction) caused by bathymetry surrounding the CTBTO stations.

6. ACKNOWLEDGEMENTS

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REFERENCES

[1] **Villán, José Luis**, The Tragic Loss of ARA San Juan, *Proceedings of the U.S. Naval Institute*, Vol. 45, pp. 1393, 2019.

[2] **Dushaw, Brian D.,** WIGWAM Reverberation Revisited, *Bulletin of the Seismological Society of America*, vol. 105, pp. 2242-2249, 2015.

[3] **Nielsen P., M. Zampolli, R. Le Bras, P. Mialle, P. Bittner, A. Poplavskiy, M. Rozhkov, G. Haralabus, E. Tomuta, R. Bell, and P. Grenard**, "Analysis of hydroacoustic and seismic signals originating from a source in the vicinity of the last known location of the Argentinian submarine ARA San Juan," *European Geosciences Union General Assembly*, Vol. 20, EGU 2018-18559, 2018.

[4] **Munk, W.H., and F. Zachariasen,** Refraction of sound by islands and seamounts, *J. Atmos. Ocean. Tech.,* vol. 8, pp. 554–574, 1991.

[5] **Collins, Michael,** A Split-Step Padé Solution for the Parabolic Equation Method, *Journal of the Acoustical Society of America,* vol. 93, pp. 1736-1742, 1993.

[6] **Porter, M. B. and E. L. Reiss,** "A numerical method for ocean acoustic normal modes," *J. Acoust. Soc. Am.* 76, pp. 244-252, 1984.

[7] **Heyser, Richard C.,** Instantaneous intensity, *Proceedings of the 81st Convention of the Audio Engineering Society* , Preprint 2399 (1986).