VERTICAL ARRAY GAIN IN SHALLOW WATER

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Abstract: *Detecting sound in the ocean is greatly improved by hydrophone array processing. Until recently, operating large arrays required cumbersome data acquisition systems and processing computers. Thanks to miniaturisation of electronics and improvements in computing power, array size is now limited by practicalities of deploying large physical apertures rather than the difficulties of the associated electronics. This allows improving signal to noise ratio, in particular against ambient noise. An important component of ambient noise is noise originating from the sea surface, which is strongly anisotropic.*

The detection performance of an array can be quantified by the directivity index which is easy to compute but difficult to measure or the array gain which is easily measurable but challenging to model. Directivity index is defined for isotropic noise, while array gain, which determines the actual performance of the system in a given environment, is defined for any noise directionality. Extending the vertical or horizontal aperture of an array may result in any an increased directivity index but array gain will also be affected by signal coherence and noise anisotropy. These two characteristics are in turn influenced by all the environmental parameters related to acoustic propagation (water column, surface and seafloor parameters).

In this paper, we use semi analytical formulas of array gain to study array gain against surface noise in simple shallow water environments. In particular we quantify the relation between array gain, aperture and geo-acoustic parameters and compare it with directivity index.

Keywords: *sonar performance modelling, vertical array*

1. INTRODUCTION

In "back-of-the envelope" sonar performance computations and manufacturers' brochures, sonar receiving array performance is often quantified in terms of directivity index (DI) instead of array gain (AG). This makes sense as the former is easy to compute as it is only a property of the sonar while the latter depends on both the environment and the source and requires more complex computations that are case-specific. In the case of a horizontal line array, DI is a good approximation for AG in most cases. However, for an array with vertical directivity (i.e. for a horizontal line array in bearings towards end-fire or an array with vertical aperture), the directionality of the signal to be detected and that of the noise have a large influence on AG.

In this paper, we concentrate on the AG of vertical line arrays, and use simple modelling to quantify its departure from DI as a function of source range and environmental parameters. The array aperture is left as a free parameters in order to assess the added value of a longer array. A similar but more in-depth analysis presenting comparable results is provided by Rachel Hamson in [1] using different types of models and comparison with measured data.

In Section 2, we recall definitions for array gain and directivity index. The detail of array gain computation in the considered environments is given in section 3. In section 4, we use the presented formula to analyse the behaviour of array gain and its departure from directivity index in a typical shallow water environment.

2. DEFINITIONS

Array gain is defined in [2] as the difference in SNR level for a single hydrophone and after array processing (beamforming):

$$AG = SNR_{hydro} - SNR_{bf}. \tag{1}$$

We express SNR level in terms of difference of sound pressure level of noise and signal:

$$SNR \; = 10 \log_{10} \frac{Q_s}{Q_n}, \tag{2}$$

where Q_n and Q_s are the mean square pressure of noise and signal respectively.

From these expression, it is clear that the source factors of noise and signal all cancel out when the difference in SNR is computed, and we will therefore omit them for the rest of the paper. The mean square pressure in a given beam after beamforming is the integral over all solid angles of all the contributions of noise or signal:

$$Q = \int_{\Omega} dQB(\varphi, \theta) \mathrm{d}\Omega = \int_0^{2\pi} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} dQB(\varphi, \theta) \cos\theta \; \mathrm{d}\theta \mathrm{d}\varphi \,, \tag{3}$$

where φ and θ are azimuth and elevation (positive upwards), respectively, B the array beampattern and dQ is the mean square pressure contribution per solid angle.

Directivity index is a special case of array gain, as it is defined as array gain for a plane wave against isotropic noise [2]. In this case, the expression of array gain simplifies to

$$AG_{iso} = DI = -10\log_{10}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\int_{0}^{2\pi} B(\varphi,\theta)\mathrm{d}\varphi\cos\theta\,\mathrm{d}\theta. \quad (4)$$

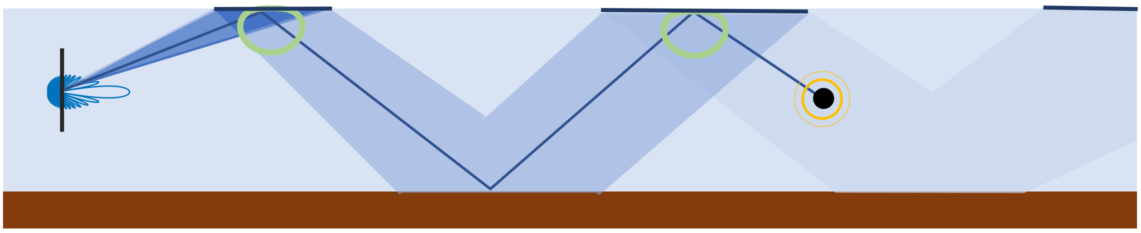
The beampattern $B(\varphi,\theta)$ (as defined in [2]) of most arrays can be readily calculated using techniques presented in [3]. In this paper, we will concentrate on the added value of the vertical aperture of arrays, and therefore restrict ourselves the properties of vertical line arrays. In the rest of the paper, we assume the beamforming carried out is the unweighted delay-and-sum beamformer. For a continuous unweighted vertical line array steered at broadside (horizontal), the beampattern is:

$$B(\varphi,\theta) = \mathrm{sinc}^2\left(\frac{2\pi L}{2\lambda}\sin\theta\right), \quad (5)$$

where λ is the wavelength, L the array length and sinc is the cardinal sine function $(\sin x/x)$.

3. ARRAY GAIN IN A SHALLOW WATER ENVIRONMENT

In this section we will recall formulas from literature for angular contributions of signal and noise in a shallow water waveguide.



3.1. Signal

Let us first consider the signal contribution. The mean square pressure contribution per solid angle to the total received signal after beamforming is,

$$dQ_S = S(\varphi,\theta)B(\varphi,\theta)F(\theta), \quad (6)$$

where $S(\varphi,\theta)$ is the source factor ($1\ \mu\mathrm{Pa}^2\mathrm{m}^2/\mathrm{ste}$ for this paper) and $F(\theta)$ is the propagation factor for a single eigenray:

$$F(\theta) = \frac{\cos^2\theta}{r^2}\exp\left(-\frac{2\alpha r}{\cos\theta}\right)|R(\theta)|^{\frac{r}{H}\tan\theta}, \quad (7)$$

where r is the source range, α the volume absorption in water, $R(\theta)$ the seafloor amplitude reflection coefficient. The exponent of the reflection coefficient accounts for the $\frac{r}{2H}\tan\theta$ reflections at the seafloor and $r/\cos\theta$ is the eigenray path length. Following [4], we compute the total mean square pressure as a continuous sum of incoherent eigenrays over all solid angles.

Computing array gain involves computing the signal mean square pressure before and after beamforming. We use omni-directional hydrophones and therefore use $B(\varphi,\theta) = 1$ to compute the mean square pressure before beamforming. While it is possible to obtain a closed form solution for this integral by making simplifications on the beampattern and the reflection coefficient [2], we choose to evaluate it numerically to allow using arbitrary beampatterns and reflection coefficients. The resulting $F(\theta)$ function is plotted in Fig. 2 (solid and dashed black lines), together with the critical angle (black mixed line) and the reflection coefficient for the chosen environment (see section 4 for environmental description).

3.2. Noise

To compute the noise contribution, we use Chapman's work [5] on noise directionality. He shows that the summed contributions of successive patches of surface dipoles radiating to the receiver within a solid angle (Fig. 1) result in an infinite series which elegantly simplifies in the following expressions:

$$dQ_N(\theta < 0) = \frac{B(\varphi,\theta)S_0 \sin\theta \, |R^2(\varphi)|}{1 - |R^2(\theta)|} \tag{8}$$

$$dQ_N(\theta > 0) = \frac{B(\varphi,\theta)S_0 \sin\theta}{1 - |R^2(\theta)|}. \tag{9}$$

where S_0 is an areic source level term disappearing in the AG computation.

In this paper, we neglect absorption, but treatment of absorption and other phenomena are treated extensively by Harrison [6][7]. The resulting noise directionality ($dQ_n(\theta)$) is plotted in Fig. 2.

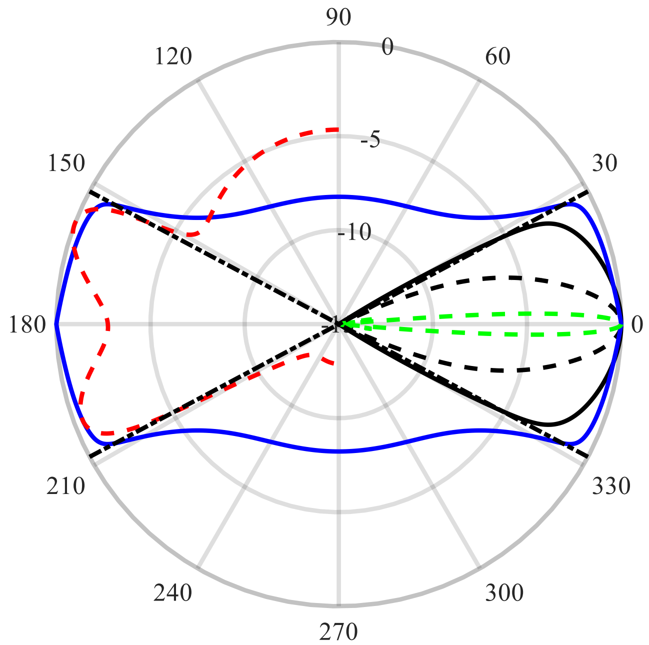


Fig.2: Red dashed line noise directivity, blue reflection gain (i.e. reflection loss but always negative so that adding the gain reduces the level), black signal directivity for a source at range 1 km (solid line), 10 km (dashed line), green dashed line, directivity pattern of a 10λ vertical array

3.3. Array Gain

The array gain is computed by integrating all angular contributions over solid angle:

$$AG \ = 10\log_{10}\left(\frac{\int_\Omega dQ_{S,bf}\mathrm{d}\Omega \int_\Omega dQ_{N,hydro}\mathrm{d}\Omega}{\int_\Omega dQ_{S,hydro}\mathrm{d}\Omega \int_\Omega dQ_{N,bf}\mathrm{d}\Omega}\right), \tag{10}$$

where the suffix *hydro* corresponds to the mean square pressure for a single hydrophone (i.e. $B(\varphi,\theta) = 1$) and *bf*, the mean square pressure after beamforming. In Fig. 2, the elements of the integrands of equation (10) are plotted in a polar format. This plot seems to indicate that all contributions are strongly affected by the critical angle and the signal directionality depends strongly on the source range. In the following section, we will therefore look at the sensitivity of array gain, for different array lengths, to these parameters. We will further look at the influence of wind as wind speed and surface roughness are usually affecting all terms of the sonar equation.

4. PARAMETER VARIATION

The basic shallow water environment used in this paper is that used for the David Weston Memorial Workshop on sonar performance modelling [8]. Additionally, some environmental parameters are varied.

4.1. Effect of source range

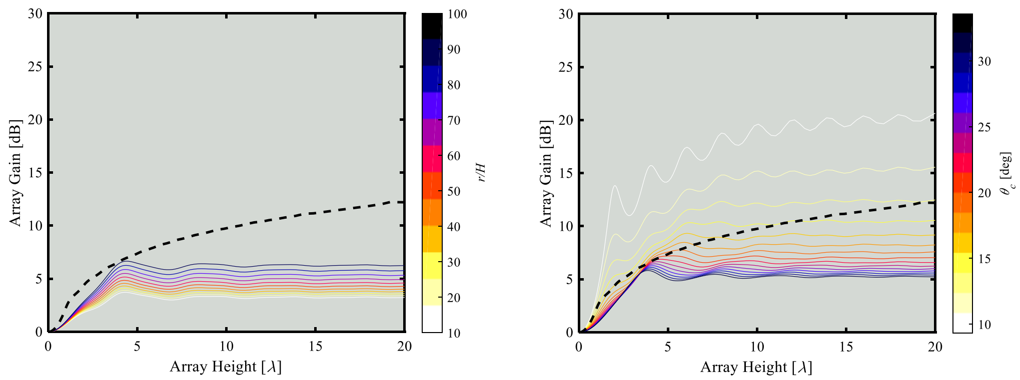


Fig.3: Left: Directivity Index (dashed line) and Array Gain as a function of target range (in number of water depths) and array height (in wave lengths). Right: same for varying seafloor critical angle.

As shown in Fig. 2, the signal contribution becomes more directional with increasing source range. We computed AG for a source range increasing from ten to one hundred times the water depth (100 m). The results are shown in the left panel of Fig. 3. The dark lines correspond to the furthest range and the bright lines to the closest range. A few observations can be made: In this case, array gain is always less than directivity index. Also, while DI keeps increasing with array length, AG saturates at about four wavelengths, whatever the source range. The saturating value depends on the source range, with a further source allowing increased AG. This can be understood by considering Fig 2.: as the source range increases, steeper paths get more attenuated and the signal is more directional around the horizontal. A longer array only over-resolves the signal resulting in no increased array gain.

4.2. Effect of critical angle

The critical angle has a more complex influence than source range, as it influences both signal and noise. In the right frame of Fig. 3, the critical angle of the sediment is varied from around 9 deg to about 33 deg (other geo-acoustic parameters such as sediment density and compressional attenuation are fixed to the values from [8]). Array gain saturates around four wavelengths for faster sediments and a bit more for slower sediments. For slower sediments, (brighter lines), array gain can even be higher than directivity index. What this implies for the performance of a sonar would require an analysis including shipping noise [9].

4.3. Effect of wind

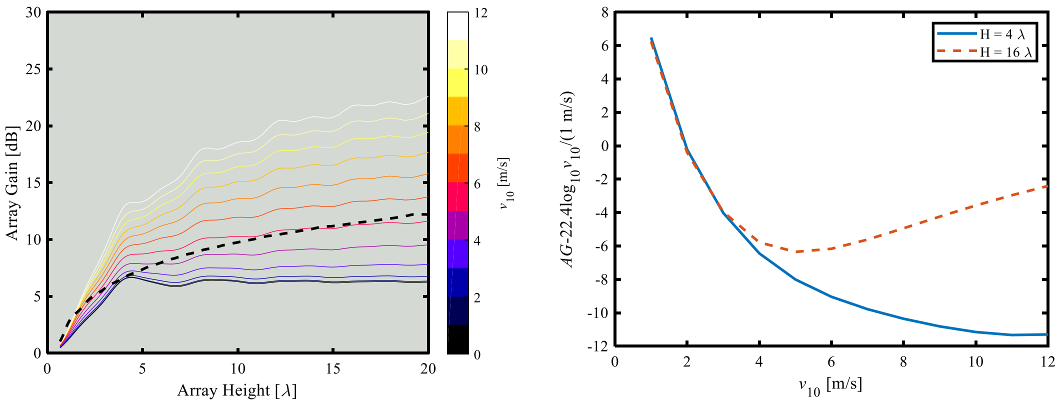


Fig.3: Left: Directivity Index (dashed line) and Array Gain as a function of wind speed and array height (in wave lengths). Right: array gain minus noise increase due to wind speed as a function of wind speed for two array heights.

We carried out a similar analysis including surface loss due to wind, using the reflection coefficient from [2], with a wind speed at an altitude of 10 m from 0 m/s to 12 m/s. As one can expect, a mechanism similar to that of the increasing critical angle is visible in the results in Fig. 3. An increase in wind strips down the higher angle paths thereby increasing the array gain. As this may suggest that detection performance increases as wind speed increases, we attempted to consider the net effect on SNR without having to compute a full sonar equation. The mean-squared pressure areic source factor due to wind-generated surface noise from [2] is proportional to $v_{10}^{2.24}$, therefore we plotted $AG - 22.4\log_{10}(v_{10}/1\text{ m/s})$ as a proxy for an hypothetical SNR, for two array lengths, in the right panel of Fig. 4. For both array lengths, an increase in wind only results in a degradation in SNR in comparison to the low wind case. For the longest array (16 λ), the SNR is increasing again beyond 5 m/s, but the resulting SNR is still small compared to a situation without wind.

5. CONCLUSION

Using simple formulae from literature we have shown how array gain can be computed for a shallow water environment. With these formula we analysed the sensibility of array gain to a few parameters (source range, sediment critical angle and wind speed) and compared it to directivity index.

This has shown that, as far as vertical aperture is concerned, directivity index is not a very good approximation of array gain. Furthermore, array gain seems to level off with increasing array length quicker than directivity index (with an array length around 4 λ) for the cases considered here. Finally, as an exercise in modelling, we have shown that while an increase in wind speed results in more array gain, it only results in an increase in SNR beyond a certain wind speed.

In general, the estimation of the performance of an array with vertical aperture requires taking into account the directionality of noise and signal to meaningfully reduce uncertainty in sonar performance prediction.

ACKNOWLEDGEMENTS

This work was sponsored by the Defence Materiel Organisation of The Netherlands. The authors are indebted to M. Ainslie and S. van IJsselmuide for useful suggestions and discussions.

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