INFLUENCE OF FLUID-STRUCTURE COUPLING ON TARGET ECHO STRENGTH SIMULATION

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Abstract: *A combined Boundary Element – Finite Element method for the calculation of the Target Echo Strength (TES) of an underwater object is presented. For the calculation of TES in the high frequency range it is not necessary to consider fluid-structure-coupling. In this case it is possible to use a raytracing based algorithm together with a Kirchhoff scattering approximation. Better results can be obtained by using a method like the BEM, that is able to include the influence of the mass of object as a thin shell. But for the low frequency region the elastic response of the object has to be considered as well. This demands the application of a full fluid-structure interaction, which can be done with a BEM-FEM coupling. Three different formulations for this coupling will be explained and compared. Results of calculations for a test object consisting of a water-filled steel cylinder with one semi-spherical end cap with the different coupling algorithms will be presented. The coupled approach makes it possible to detect TES highlights at several frequencies that are caused by eigenmodes. It will be shown that only some eigenmodes are able to radiate. The presented BEM-FEM approach is able to calculate the TES of models consisting of more than a million elements.*

1.1. ***Keywords:*** *Target Echo Strength, FEM, BEM, coupling*

1. INTRODUCTION

Target echo strength is a quantity that describes the backscattered energy of an object in water in the far field, excited by an incident acoustic wave. The physically most precise method to calculate this is the finite element method (FEM) that takes the fluid structure coupling of the object with the surrounding water into account. The problem with this approach is that the computational load gets enormous quickly for three dimensional calculations because the full 3D domain including the water has to be discretised. A usual method to circumnavigate this problem is using the boundary element method (BEM). The equations are derived from the Helmholtz equation, cast as integral equation in the weak form, by application of the Gauss integral theorem. The advantage of the BEM is that for the resulting equations only the surface of the object and not the fluid domain has to discretised. This reduces the problem to a two-dimensional one which. The disadvantage is that both the structural response of the object to the excitation and the fluid structure coupling are neglected. As one way to remedy the latter at least partially for thin structures the so-called mass inertia coupling can be used. Thereby the mass of the thin structure gives an inertia boundary condition for the coupling of the fluids on the front and the back of the object [1]. In order to also consider the response of the excited structure itself and its effect on the fluid a FEM method has to be used. This is especially important at low frequencies where discrete resonances of the structure are important. In order to retain the advantage of the BEM not having to discretise the 3D fluid domain a coupling between FEM and BEM has to be made. Foundations for FEM and BEM for acoustics can be found in [2]. In the following such a method is proposed for structures that can be modelled with thin shells and three approaches for the solution of the resulting matrix equations are explained. Calculations for a test structure in water are presented.

2. FLUID STRUCTURE COUPLING

The presented approach for this task is based on an already existing BEM code that can handle direct and indirect BEM formulations. It uses a BEM collocation formulation with constant basis functions and triangular elements. For the indirect BEM formulation which is appropriate for thin structures, the unknowns are the acoustic normal velocity $\vec{v}$ and the step of pressure δp between the front and the back of the object. For constant elements the unknowns are placed in the centre of the object.

For the FEM description of the object a formulation is chosen that uses Love-Kirchhoff plate theory. This means that the object is not discretized with three-dimensional solid elements but with two-dimensional shell elements which leads to the restriction to objects with thin structures. For these elements the unknowns are the translational degrees of freedom w_x, w_y, w_z and the rotational degrees of freedom φ_x, φ_y and φ_z. These unknowns are placed on the corners of the triangular elements. Moreover, the discretisation of the object is not necessarily the same for FEM and BEM. In order to apply the fluid structure coupling in this case a mortar coupling is used. The idea is to distribute the loads onto the other DOFs taking into account the geometric positions of the respective nodes and the element size.

The equation governing the FEM model is

$$(\mathbf{K} - \omega^2 \mathbf{M}) \begin{bmatrix} \vec{\varphi} \\ \vec{w} \end{bmatrix} = \mathbf{K}_f \begin{bmatrix} \vec{\varphi} \\ \vec{w} \end{bmatrix} = \mathbf{K}_f \, \vec{\boldsymbol{x}} = \begin{bmatrix} \mathbf{K}_{\varphi\varphi} & \mathbf{K}_{\varphi w} \\ \mathbf{K}_{w\varphi} & \mathbf{K}_{ww} \end{bmatrix} \begin{bmatrix} \vec{\varphi} \\ \vec{w} \end{bmatrix} = \begin{bmatrix} 0 \\ \vec{f} \end{bmatrix} \quad (1)$$

with the mass matrix $\mathbf{M}$ and the stiffness matrix $\mathbf{K}$ that give the resulting matrix $\mathbf{K}_f$, the excitation force $\vec{f}$. The matrix $\mathbf{K}_f$ can be split into four submatrices that either describe the coupling of translational DOFs and rotational DOFs or the cross couplings. The matrices are sparse, as it is usual for FEM matrices.

The indirect BEM equation is given by

$$-\rho c \cdot \overrightarrow{v_n} + \boldsymbol{J}_B \cdot \overrightarrow{\delta v_n} + \boldsymbol{F}_B \cdot \overrightarrow{\delta p} = -\rho c \, \vec{v}_{inc} \tag{2}$$

with the variables normal velocity $\overrightarrow{v_n} = (\vec{v}_x, \vec{v}_y, \vec{v}_z)$, step of pressure between front and back $\overrightarrow{\delta p}$ (double layer potential) and step of velocity $\overrightarrow{\delta v_n}$. The latter is assumed to be zero since in the shell formulation the velocity is equal on both sides. Therefore, the equation simplifies to

$$-\rho c \cdot \overrightarrow{v_n} + \boldsymbol{F}_B \cdot \overrightarrow{\delta p} = -\rho c \, \vec{v}_{inc} \tag{3}$$

where $\boldsymbol{F}_B$ is the fully populated BEM matrix. The coupling of FEM and BEM is done by linking the force term $\vec{f}$ of the FEM to the jump of pressure $\overrightarrow{\delta p}$ of the BEM and the translations $\vec{w}$ to the velocity $\vec{v}$ via coupling matrices $\mathbf{T}_1$ and $\mathbf{T}_2$ as

$$\vec{v} = \mathbf{T}_1 \cdot \vec{w} \quad \text{and} \quad \vec{f} = \mathbf{T}_2 \cdot \Delta\vec{p} \tag{4}$$

This leads to the coupled matrix equation

$$\begin{bmatrix} \mathbf{K}_{\varphi\varphi} & \mathbf{K}_{\varphi w} & 0 \\ \mathbf{K}_{w\varphi} & \mathbf{K}_{ww} & \mathbf{T}_2 \\ 0 & -\rho c \mathbf{T}_1 & \mathbf{F}_B \end{bmatrix} \begin{bmatrix} \vec{\varphi} \\ \vec{w} \\ \overrightarrow{\delta p} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -\rho c \, \vec{v}_{inc} \end{bmatrix} \tag{5}$$

or

$$\begin{bmatrix} \mathbf{K}_f & \mathbf{T}'_2 \\ -\rho c \mathbf{T}'_1 & \mathbf{F}_B \end{bmatrix} \begin{bmatrix} \vec{\mathrm{x}} \\ \overrightarrow{\delta p} \end{bmatrix} = \begin{bmatrix} 0 \\ -\rho c \, \vec{v}_{inc} \end{bmatrix} \tag{6}$$

with the shorter notation of $\vec{x} = [\vec{\varphi}, \vec{w}]^T$ from equation (1). The coupling matrices $\mathbf{T}_1$ and $\mathbf{T}_2$ are replaced with $\mathbf{T}'_1 = [0, \mathbf{T}_1]$ and $\mathbf{T}'_2 = [0, \mathbf{T}_2]^{\mathrm{T}}$.

3. SOLUTION STRATEGIES

Equation (6) is a matrix equation with the left hand matrix consisting of the large sparse FEM matrix $\mathbf{K}_f$, the smaller but fully populated BEM matrix $\mathbf{F}_B$ and two sparse coupling matrices $\mathbf{T}'_1$ and $\mathbf{T}'_2$. There are several possible methods to solve this equation.

The straightforward way is a direct solution of the matrix equation (6). The resulting matrix is very large und needs a lot of memory for the decomposition. It is also not easy to solve since it consists of a sparse part and a fully populated part. Treating the whole matrix as a sparse matrix demands a lot of memory.

An alternative method uses the Schur approach. This can be done by inverting the upper equation of (6) to $\vec{x} = -\mathbf{K}_f^{-1} \, \mathbf{T}'_2 \cdot \overrightarrow{\partial p}$ (Schur complement) and inserting it into the lower equation. This leads to the Schur equation

$$\rho c \mathbf{T'}_1 \mathbf{K}_f^{-1}\, \mathbf{T'}_2 \cdot \overrightarrow{\partial p} + \mathbf{F}_B \cdot \overrightarrow{\partial p} \; = -\rho c\, \vec{v}_{inc} \tag{7}$$

The difficult part would be the inversion of the large FEM matrix $\mathbf{K}_f$. But by using an iterative solution strategy it can be avoided to do the matrix inversion explicitly. Instead only matrix vector products with $\mathbf{K}_f$ have to be calculated. But this method still uses the large FEM matrix.

A way to reduce the effective size of the matrix can be found by using the eigenmodes of the uncoupled FEM equation (dry modes). A given number of the lowest eigenmodes can be extracted from $\mathbf{K}_f$ with modest numerical effort. These correspond to the resonances of the structure in vacuum. With the eigenvectors a matrix $\boldsymbol{\psi}$ can be formed that has the dimension number of DOFs times number of eigenvectors (N_{DOF} x N_{ev}), in contrast to the square matrix $\mathbf{K}_f$ which has the dimension N_{DOF} x N_{DOF}. With this matrix $\boldsymbol{\psi}$ a transformation of $\mathbf{K}_f$ can be made that results in transformed matrices $\mathbf{H}_{ev}$ with the reduced size of N_{ev} x N_{ev}

$$\mathbf{H}_{ev} = \mathbf{\Psi}_{ev}^T \cdot \mathbf{K}_f \cdot \mathbf{\Psi}_{ev} \tag{8}$$

$\mathbf{H}_{ev}$ is diagonal and small and can be inverted much more easily than $\mathbf{K}_f$. This leads to the final equation for the modal approach

$$(-\rho c\, \mathbf{T'}_1\, \mathbf{\Psi}_{ev}\, \mathbf{H}_{ev}^{-1}\, \mathbf{\Psi}_{ev}^T\;\, \mathbf{T'}_2 + \mathbf{F}_B) \cdot \overrightarrow{\partial p} \; = -\rho c\, \vec{v}_{inc} \tag{9}$$

The physical meaning of this approximation is that all results for the coupled equation are linear combinations of the modes, i.e. the vibration patterns of the uncoupled system.

All three equations (6), (7) and (9) have a similar structure with a matrix times the solution vector $\overrightarrow{\partial p}$ on the left hand side and the incident sound velocity $\vec{v}_{inc}$ on the right hand side. Table 1 gives an overview of the proposed solution methods, its characteristics and applicability.

Method	*BEM*	*Sparse*	*Schur*	*Modes*
Memory size	small	large	large	small
Realistic results	+	++	++	++
Solver	direct iterative	direct	iterative	direct iterative
Frequency regime	medium high	low	low medium	low medium
Fast multipole method	yes	no	yes	yes

Table 1: Solution methods for TES calculations and its characteristics

4. EXAMPLE

In order to examine the procedures a test body was devised. It consists of a half open steel cylinder with the thickness of 6 mm with a hemispherical end cap of the same thickness on one side. The other side is open. The cylinder has a diameter of 0.5 m and the whole structure is 1 m long and immersed in water.

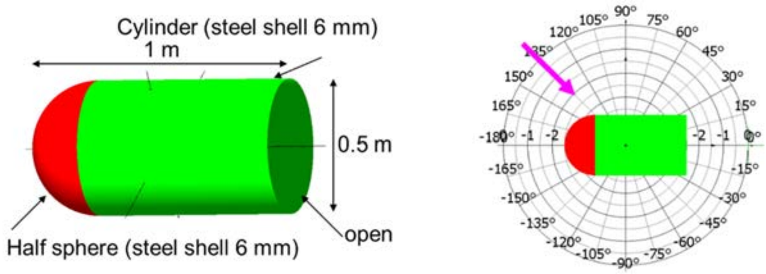


Figure 1: Test structure with open cylinder and hemispherical end cap (left) and direction of insonification.

The target echo strength of this test body was calculated for insonification from 135° aspect angle and 0° elevation angle using the above methods for a frequency range from 300 Hz to 1000 Hz. As a reference TES was calculated with the commercial FEM code Comsol in a fully coupled 3D calculation. Besides the TES calculation, the eigenmodes of the uncoupled und the coupled structure in water were calculated.

5. RESULTS

The problem could not be solved by solving equation (6) with a direct solver for sparse matrices due to memory and time restrictions. The Schur approach and the eigenmode approach were used for the calculation of TES, the latter with different numbers of dry eigenmodes used for the expansion. Fig. 2 shows the results for the calculations compared with the Comsol FEM calculation.

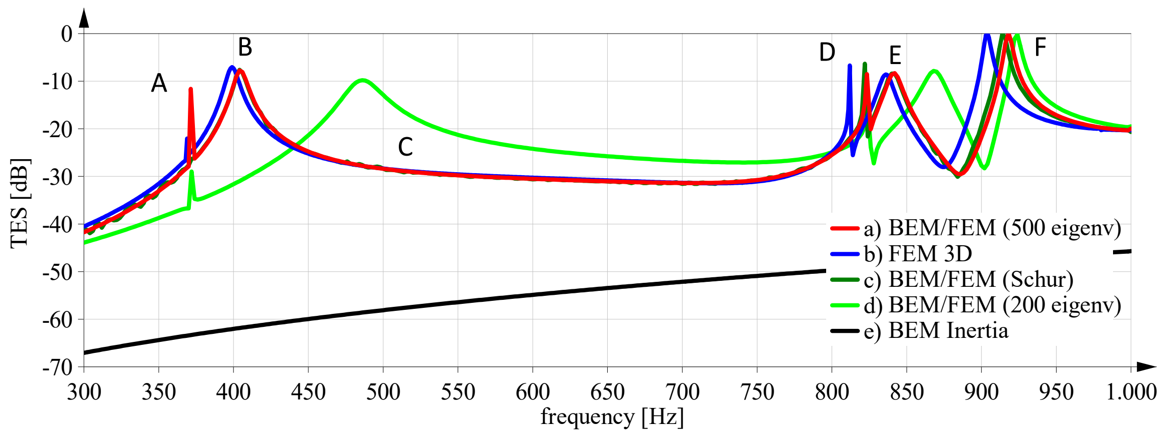


Figure 2: TES calculation results for the test structure with FEM 3D calcualtion, FEM/BEM coupling with Schur and modes and BEM inertia coupling.

Several things can be seen in the graph. The blue Comsol result can be considered to be the reference. All results of methods with full coupling show a broadly similar shape. The curves all have a few distinct sharp and broad peaks that correspond to resonances, where the vibration of the structure and, hence, backscatter is stronger. The BEM inertia result (black curve) is incorrect in this frequency range and underestimates TES by at least 20 dB. The result of the mode calculation with 200 eigenmodes (green curve) is also insufficient. The resonance peaks are at incorrect frequencies. The results of the Schur calculation and the mode calculation with 500 eigenmodes are very similar to each other and differ only slightly from the Comsol result. This proves the success of the coupled BEM/FEM calculations.

For the eigenmode approach, which is the fastest of the three, the number of eigenmodes has to be considerably higher than 200 for this example and the frequency range considered. The discrepancy of the results from the Comsol result are probably due to the fact that we used shell elements from Love-Kirchhoff plate theory whereas Comsol used Reissner-Mindlin shell elements. The difference between these formulations requires further investigation.

All results show a relatively smooth curve with a few distinct peaks at resonances. But when you calculate the eigenmodes of the coupled problem, you find a much greater number of modes in the frequency range considered that do not contribute visibly to the TES curve. These modes are obviously not radiating. Comsol gave the ability to produce pictures of the eigenmodes found for the dry modes. Table 2 shows a selected number of eigenfrequencies for the dry case (in vacuum), the wet case and the case of a rigid structure in water for the corresponding modes. For important modes pictures of vibrating shape of the modes are shown above and labelled. The letters correspond to the ones in Fig. 1. For the dry modes the number of the mode is added in parentheses.

For all modes the frequency is lower in the coupled case due to the fluid loading of the structure. This phenomenon is well known. These are the only modes where the eigenfrequency has a non-negligible imaginary part. This reflects the radiation into the far-field of these modes which corresponds to a loss of energy of the mode and hence an imaginary part. The next similarity of these modes is that their mode shape shows a larger distance between maxima and minima than for the other modes. One example of a non-radiating mode is C at 946 Hz dry / 516 Hz wet. If you attribute a wavelength to the mode shapes defined the longitudinal difference between maxima and minima on the structure and compare it to the acoustic wavelength in water you find that the mode wavelength is in the order of the water wavelength or larger for the radiating modes and much smaller for the non-radiating modes.

There are two radiating modes that have no equivalent in the dry calculation but in the sound hard calculation. The can be seen as a kind of Helmholtz resonators that are not present in the dry calculation. They roughly correspond to a quarter water wavelength and a half water wavelength fitting in the length of the structure. In the dry (in vacuum) calculation this condition is not met since there is no medium with an acoustical wavelength present.

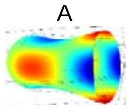
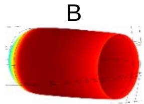
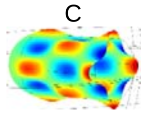
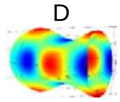
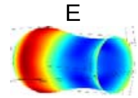
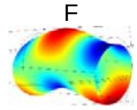
		A 	B 	C 		D 	E 	F 
dry mode	71 (7)	**780 (21)**		**946 (27)**	1394	**1704 (53)**		**1780 (55)**
wet mode	32	**369+0.01i**	**399+4i**	**516**	845	**812+0.2i**	**836+5i**	**903+2i**
rigid			**354+5i**				**1070+50i**	

Table 2: Eigenfrequencies of selected modes in the dry, wet and rigid calculation in Hz and mode shapes above. The order of the dry modes is added in parentheses

The monostatic TES calculation was repeated for other all aspect angles from 0° to 180° and frequencies from 100 Hz to 1 kHz. The results in Fig. 3 show the magnitude of the calculated TES over the aspect angles. The magnitude distribution of peaks reveals the principal vibration shapes of the resonances. The first radiating mode A has a dipole characteristic with a main direction around 90°. The broader Helmholtz resonator mode B also has this pattern. The same applies to the next sharp peak D. The second order Helmholtz mode E, however, has a minimum at broadside and maxima at the front and the back. The first bending mode F has its main radiation direction at 45°/135°.

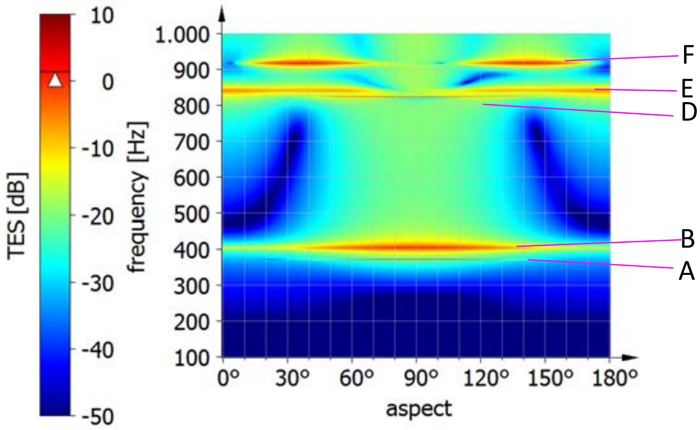


Figure 3: Monostatic TES calculation over frequency for aspect angles from 0° to 180°.

6. SUMMARY AND OUTLOOK

A coupled FEM / BEM method for the calculation of target echo strength of thin structures in the low frequency regime was described. Three methods for the solution of the coupled matrix equations were presented. The direct solution is hardly practicable because the matrix gets too big. The Schur method improves on that and the mode leads to the smallest matrix to solve. Calculations for a test structure were presented and compared to results of a finite element calculation with a commercial code. The results of the Schur method and the modal decomposition agreed fairly well with the results of the FEM calculation. Remaining discrepancies of resonance frequencies are probably due to the different FEM shell element types that were used.

The mode calculations revealed that although there is vast number of modes in the frequency range considered only a few of them are radiating modes and dominate target echo strength. The mode calculations show that it was not sufficient to include only 200 dry modes. The question arises whether all dry modes have to be used for the modal expansion or if it is sufficient to include only radiating modes. After knowing which modes are radiating, subsequent calculations with these modes and the lowest modes indicate that this seems to be the case. In order to automate this approach, it would be necessary to find a criterion for the dry modes to determine whether it would be radiating or not in water. This idea will be pursued in future.

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