Estimation of P- and S-wave speeds from scattering experiments on spherical samples

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Abstract: *The P- and S-wave speeds of two rubber-like materials (natural rubber and Ren-Shape) are estimated by inverse estimation from data obtained from scattering experiments on spherical samples. The samples, with a diameter of 0.15 m, are submerged at a depth of 2 m in an* $4 \times 4 \times 8$ m^3 *(length, width, depth) tank. The transmitter and receiver are positioned close to the sample using an asymmetrical triangular configuration. The transmitter-to-receiver and transmitter-to-sample distances are approximately 0.6 m and 1.2 m respectively. The configuration was optimized for improving the time separation of scattered signals. The measured scattered field from the spherical sample is matched with the field computed by a wave propagation code. By varying the input parameters (P- and S-wave speeds together with associated attenuations) to the code, the difference between measured and modelled data is minimized. This inversion procedure is performed using the global optimization algorithm differential evolution, which is well suited for finding the minimum of the resulting non-linear, non-convex optimization problem. The method in general gives good results for the wave speeds that are reasonably close to that of water. For the determination of the S-wave speed in the natural rubber sample, the relatively slow propagation prohibits a reliable estimation.*

Keywords: *Material characterization, visco-elastic, scattering, sphere.*

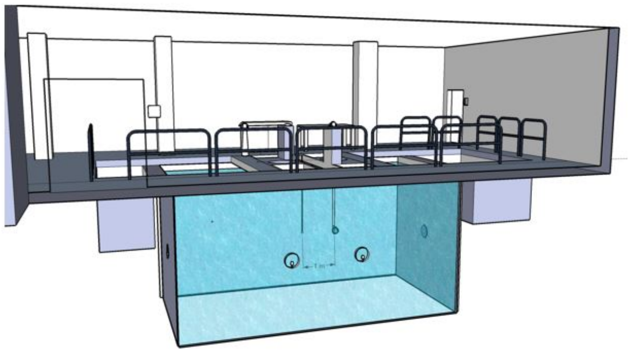


Figure 1: A schematic of measurement tank facility.

1. INTRODUCTION

The task of obtaining the visco-elastic material for rubber and foam materials is of interest for a large range of applications. Numerous methods for estimating these exist, most relying on different kinds of DMA-analysis methods. Here, a different approach is proposed; by recording the scattered signal from submerged spherical samples, the viscoelastic material parameters are estimated. This is done by mininmizing the mismatch between the recorded signals and a forward model using the global optimization algorithm Differential Evolution [1]. The method shows promising the results, giving reliable estimates for especially the P-wave speed. The method is inspired by the work conducted by Tesei *et al.* [2] and has previously been described in Ref. [3].

2. METHOD

Below follows a description of the measurement setup, data processing and inversion procedure used to obtain estimations of the visco-elastic material parameters of the studied spherical samples.

2.1. MEASUREMENTS

Two spherical samples were used in the measurements of material parameters. Sphere 1 is made of Natural Rubber and Sphere 2 is made of Renshape. The diameters of both spheres are 150 mm.

The measurements were conducted in the FOI watertank with dimensions (depth, width, length) $4 \times 4 \times 8$ m^3 (see Figure 1 for a schematic of the tank facility). Video cameras behind windows in the tank walls at 2 m depth allow for a precise positioning of the spheres and measurement equipment. The estimated distance errors in the tank are approximately 5 mm.

The spheres were suspended by attaching a thin nylon fishing line to the sphere. A 2 mm deep hole was made in the spheres using a needle. The fishing line was attached in the hole with epoxy glue. An ITC-1042 transmitter was used for the acoustic signal generation and a Brüel and Kjaer 8104 hydrophone was used as a receiver for measuring the reflected signals. The

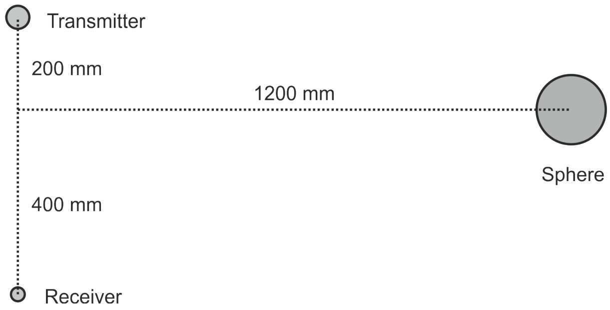


Figure 2: A schematic of the measurement configuration.

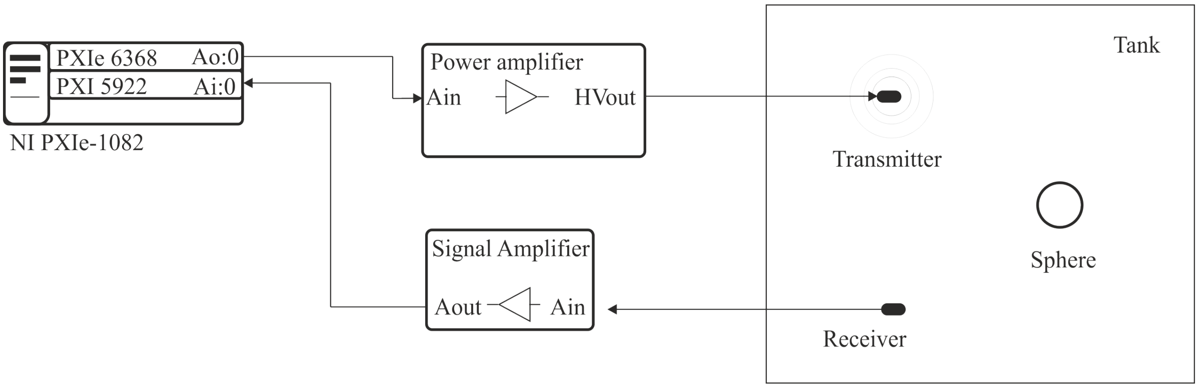


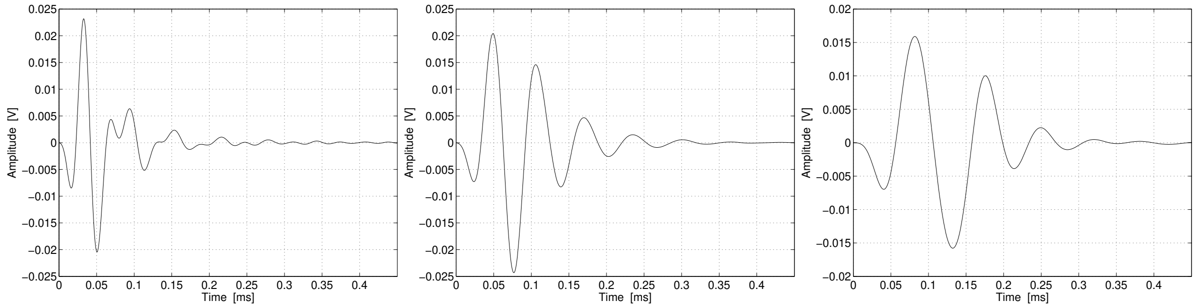
Figure 3: A schematic of the instrument setup.

transmitter, receiver and sphere setup was arranged according to Figure 2. The equipment was hanging at 2 m depth from horizontally movable bridges above the water surface, with relative distances between the transmitter, receiver and sample according to Figure 2. The configuration was optimized for optimal time gating of scattered signals from the sphere before the arrival of reflection from e.g. the surface.

The configuration of the instrument setup is shown in Figure 3. The signal generation and data acquisition is controlled using a PXIe-1082 chassi unit with a controller (built in PC). A separate PXIe 6368 card is inserted in the chassi for signal generation. The resolution of the generated signal is 16-bits with a sampling speed of 2 MS/s. The signal output (Ao:0) is connected vi a power amplifier to the ITC-1042 transmitter. The Brüel and Kjaer 8104 receiver is attached to a SR560 signal amplifier using 40 dB gain. The output of the amplifier is in turn connected to a PXI 5922 flex resolution acquisition card in the chassi. The sampling speed was set to 2 MS/s yielding a 20 bit resolution of the received signal.

An acoustical signal consisting of one period of a sine waveform was generated in the output of the PXIe 6368 card. Three different frequencies were used in this experiment; 8, 16 and 32 kHz. The Peak-to-Peak output level (HVout) from the power amplifier could reach approximately 500 V during these runs. One signal was repeated 100 times for improving the Signal-to-Noise ratio in the measurements. The repetition frequency was slightly higher than 1 Hz allowing for the echo to dissipate before the next signal generation. The reason for not using exactly 1 Hz repetition frequency is for reducing the effect from periodic noise sources. The generated sine wave signal is distorted as it is transmitted in to the water. These emanate mainly due to the “turn on transients” of the transmitter (see Figure 4 for examples).

One complete measurement set for one sphere consists of three measurement steps:



(a) 32 kHz *(b) 16 kHz* *(c) 8 kHz*

Figure 4: Incident pulses at the target position.

1. Scattering measurement with sphere, as depicted in Figure 2.
2. Background measurement. The sphere is removed from the water and the measurement is repeated.
3. Incidence measurement. The receiver is placed in the position of the sphere and the measurement is repeated.

2.2. INVERSION SETUP

In order to estimate the material parameters of the elastic spherical samples, the measures scattered signals are matched to signals obtained from a computational forward model. The model gives the scattered field from an elastic sphere in free space in terms of a series expansion of spherical Bessel functions and spherical harmonics [4]. The values of the material parameters are estimated by minimizing a cost function that quantifies the mismatch between the measured and modelled data. This results in a non-convex, non-linear optimization problem. This optimization problem is solved using a differential evolution (DE) algorithm [1]. DE is related to genetic algorithms, but the parameters are not encoded in bit strings, and genetic operators such as crossover and mutation are replaced by algebraic operators. Global optimization methods like DE are useful for nonlinear problems with non-differentiable objective functions, since the risk of getting trapped in local minima is reduced.

The inversion parameters along with their respective search interval is shown in Table 1; v_{p} is the P-wave speed, $v_{\mathrm{s}} = qv_{\mathrm{p}}$ is the S-wave speed, α_{s} is the S-wave absorption, and $\alpha_{\mathrm{p}} = \alpha_{\mathrm{p}}^{\min} + \Delta\alpha_{\mathrm{p}}$ is the P-wave absorption, where $\alpha_{\mathrm{p}}^{\min} = \alpha_{\mathrm{p}}^{\min}(v_{\mathrm{p}}, v_{\mathrm{s}}, \alpha_{\mathrm{s}})$ is the smallest value of the P-wave absorption consistent with the physical constraint for the bulk modulus. In addition to these viscoelastic parameters, amplitude and time-shift corrections, (A_{corr} and t_{shift}) are introduced to account for uncertainties in hydrophone and receiver positions. The used limits are equivalent to relative distance deviations of ± 7.5 cm.

The cost function to be minimized is the normalized rms-difference between modelled and measured data,

$$F(d,m) = \frac{\sqrt{\dfrac{1}{N}\displaystyle\sum_{i=1}^{N}\left(d(i) - m(i)\right)^2}}{\sqrt{\dfrac{1}{N}\displaystyle\sum_{i=1}^{N} d(i)^2}}, \tag{1}$$

Parameter	**Inversion constraints**	
	Natural Rubber	Renshape
v_{p} [km/s]	$[1.3, 2.0]$	$[0.05, 4.0]$
α_{s} [dB/λ]	$[0, 25]$	$[0, 25]$
q	$[0.03, 0.4]$	$[0.2, 0.7]$
$\Delta\alpha_{\text{p}}$	$[0, 10]$	$[0, 10]$
t_{shift} [ms]	± 0.05	± 0.05
A_{corr}	$[0.8, 1.25]$	$[0.8, 1.25]$

Table 1: Inversion parameters and their constraints.

d and m are vectors $[N \times 1]$ contains the measured and modelled data, respectively. The length of the data vectors is limited by the time when unwanted reflections from the tank walls/surface arrive at the receiver. In practice, this time is approximately 2.56 ms.

The DE-inversion algorithm, a population size of 90, each with 200 iterations, were completed, resulting in 18090 forward model executions. A mutation scaling factor of 0.8 and a crossover probability of 0.5 were used.

3. RESULTS

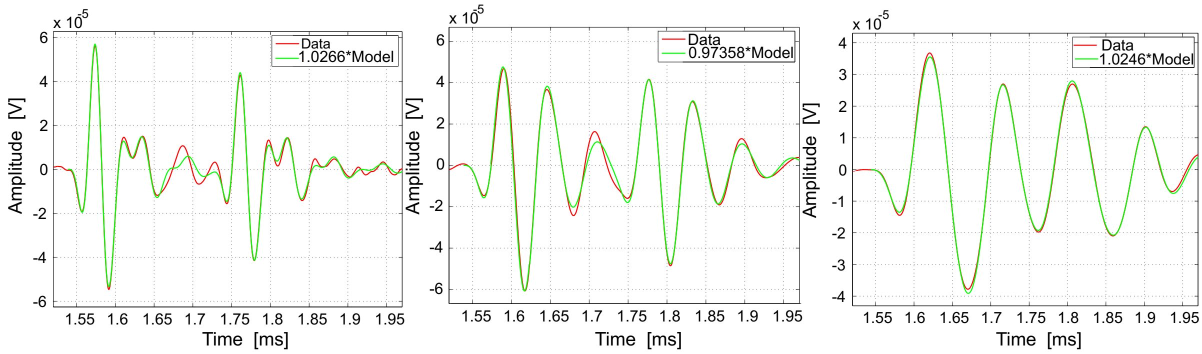
The results obtained from the inversions are presented in three ways; tabulated optimum parameters are given along with plots showing the measured data together with modelled data using the optimum parameters. Lastly, scatter plots of the objective function for all parameter combinations tested by the inversion algorithm projected onto each inversion parameter is shown. Each dot in these plots represents a forward model run.

3.1. NATURAL RUBBER SPHERE

Parameter	**NR sphere (B)**		
	32 kHz	16 kHz	8 kHz
v_{p} [km/s]	1.56	1.57	1.57
v_{s} [km/s]	0.05	0.22	0.11
α_{p} [dB/λ]	0.27	0.30	0.26
α_{s} [dB/λ]	0.01	12.6	0

Table 2: Optimal viscoelastic material parameters obtained from the inversions of the Natural Rubber sphere data. The blue color indicates that the obtained parameter value is unreliable.

In Figure 5 the recorded data together with modelled data using the optimal parameters in Table 2 is shown. It is clear that the forward model data mimics the recorded data well. From the scatter plots it is evident that the P-wave parameters, especially v_{p}, are well determined, as they exhibit sharp troughs when plotted versus the objective function. The S-wave parameters, on the other hand, are unreliable. This is likely because of the low velocity and/or high damping of reflections caused by S-waves as these would either appear outside the observed time window



(a) 32 kHz *(b) 16 kHz* *(c) 8 kHz*

Figure 5: Comparison between data and model for the optimal parameter sets (Table 2) obtained from the inversions of the Natural Rubber sphere data.

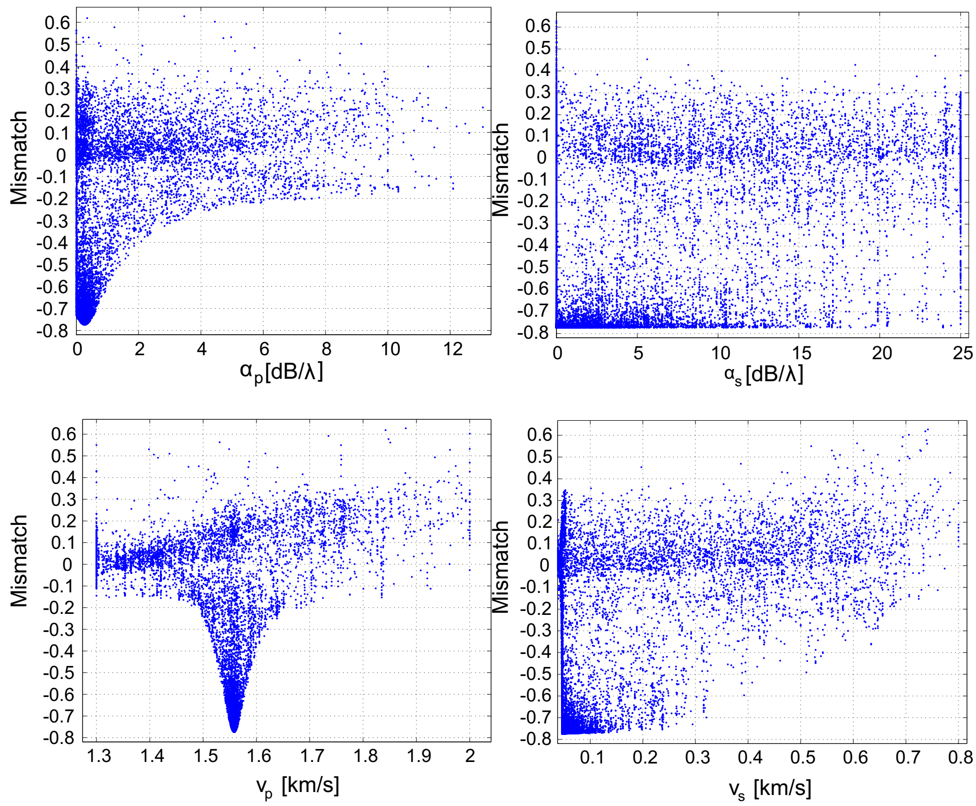


Figure 6: Scatter plots showing the objective function versus each of the sought visco-elastic parameters for the Natural Rubber sphere at 32 kHz.

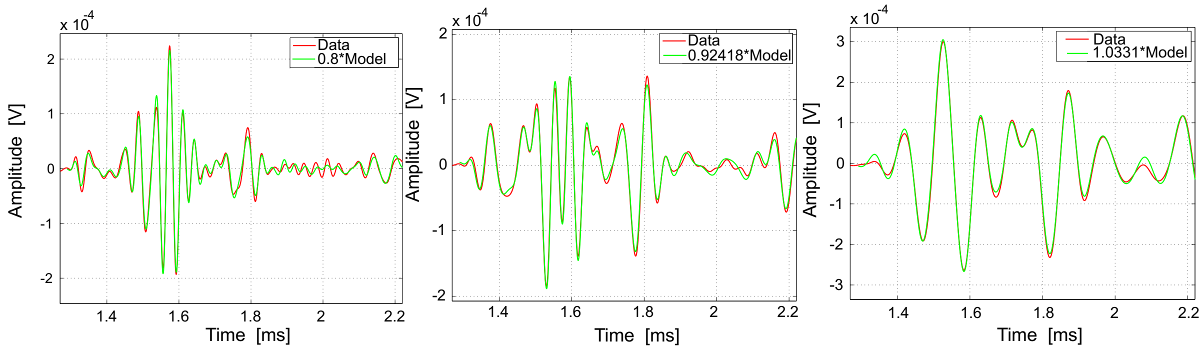
or be of negligible amplitude compared to the P-wave reflections.

3.2. RENSHAPE SPHERE

The recordings of the reflected field from the Renshape sphere used in the inversions are shown in Figure 7 along with modelled data obtained using the optimum parameters given in Table 3. The initial front edge reflection at $t = 1.30$ ms is clearly weaker than the rest of the

Parameter	Renshape sphere		
	32 kHz	16 kHz	8 kHz
v_p [km/s]	2.29	2.32	2.30
v_s [km/s]	1.23	1.23	1.22
α_p [dB/λ]	0.11	0.44	1.11
α_s [dB/λ]	0.27	0.35	0.36

Table 3: Optimal viscoelastic material parameters obtained from the inversions of the Renshape sphere data.



(a) 32 kHz *(b) 16 kHz* *(c) 8 kHz*

Figure 7: Comparison between data and model for the optimal visco-elastic parameter sets obtained from the inversions (Table 3) of the Renshape sphere data.

signal. This inidicates that most of the energy is transmitted into the sphere. Moreover, both the P- and S-wave parameters was possible to extract with good confidence, as indicated by the scatter plots in Figure 8.

4. CONCLUSIONS

The proposed method for obtaining the visco-elastic material parameters shows promising results, especially for establishing the P-wave parameters.

5. ACKNOWLEDGEMENTS

A sincere thank you to Netherlands' Defense Materiel Organization, Commit (formerly DMO) and Defence Research and Development Canada (DRDC) for contributing with manufacturing the spherical samples.

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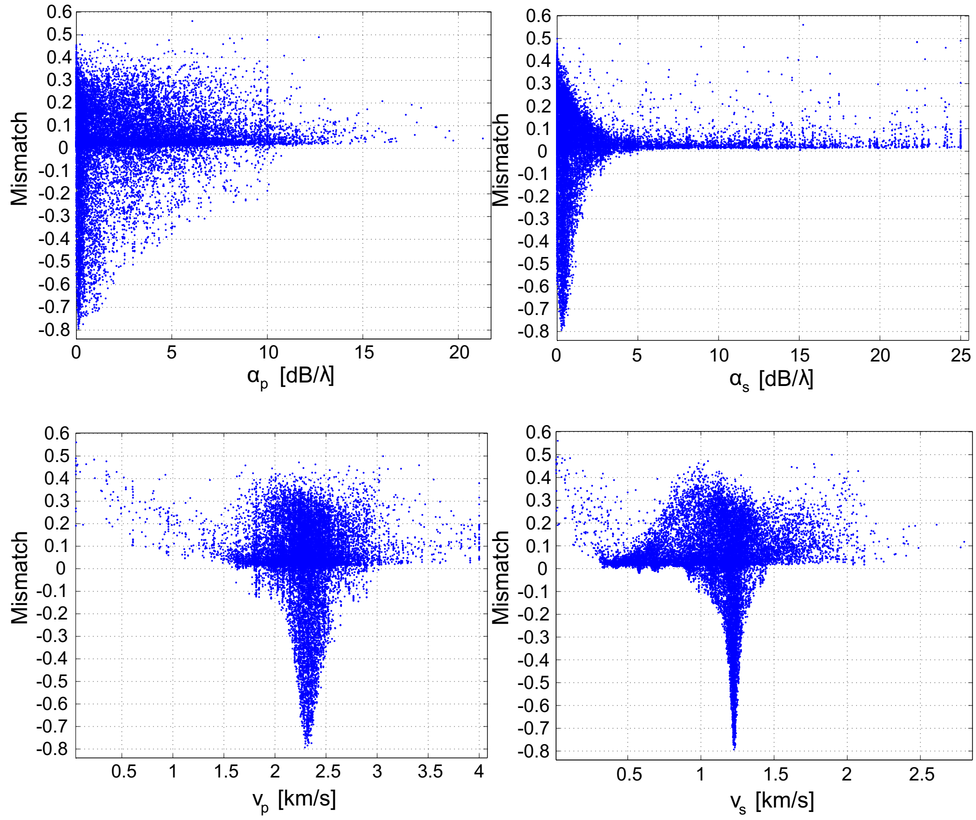

Figure 8: Scatter plots showing the objective function versus each of the sought visco-elastic parameters for the Renshape sphere at 32 kHz.

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