Using open FEM/BEM software for modeling fish target strength

Marek Moszyński[1], Isabel Pérez Arjona[2], Víctor Espinosa Roselló[3], and Anderson Ladino Velásquez[4]

[1]Gdańsk University of Technology, marek.moszynski@pg.edu.pl
[2]Universidad Politécnica de Valencia, iparjona@upvnet.upv.es
[3]Universidad Politécnica de Valencia, vespinos@fis.upv.es
[4]Universidad Politécnica de Valencia, anlave@doctor.upv.es

Marek Moszyński, ul. Gabriela Narutowicza 11/12, 80-233 Gdańsk, Poland, marek.moszynski@pg.edu.pl

Abstract: *The advance in computer modeling techniques observed in last decades allows for creating so-called computer twins in variety of real-world applications. One of the well-developed technique that found to be extremely useful in variety of acoustic areas is based on using Finite Element Method and Boundary Element Method. Both has been already commercialized in the form of advanced software packages that facilitate verification of whole designs. However in many research areas a ready to use commercial software are still not prepared for research calculations. Hence the idea of using programmable methods that allows for the insight into internal workings of the computer model with a usage of a methods dedicated for Finite/Boundary Element approach. Among many of long-developed open-source packages are FreeFEM and Bempp that is characterized by maturity in implementing of sophisticated numerical algorithms and giving higher abstraction level in software programming than modern object oriented languages. To illustrate the proposed idea the modeling of fish target strength pattern is presented. The examples are mainly oriented for using it in statistical calculation of target strength vs. fish length relation.*

Keywords: *Finite Element Method, Boundary Element Method, FreeFEM, Bempp, fish target strength, yellowfin tuna, mackerel*

1. BACKGROUND

The Boundary Element Method (BEM) has been widely used to predict scattering by fish. Among its advantages, is that this method can handle complex geometries and the solution space is one dimension lower than that of the scatterer object since field values over surfaces instead of field values over volumes are required [2]. Several software packages support BEM calculations. Over the past decade, while commercial solutions have become well-established, open-source alternatives have also matured significantly. In the paper two programmable solutions were verified using scattering by fish as a main application area, namely FreeFEM [3] and Bempp [1]. The first one, is interfaced with the boundary element library BEMTool, which is a general purpose header-only C++ library. It also uses modern technique based on representing matrices in the form of compressed hierarchical matrix stored in specialized H-matrix format. The second one is an open-source computational boundary element platform with Python interface, that uses PyOpenCL for just-in-time compilation of BEM kernels. It allows for easy formulation of acoustic and electromagnetic transmission problems, what is especially valuable when applying for fish scattering.

Modelling of the acoustic scattering from a single fish is a problem which has been extensively studied over the past decades. Since the swimbladder plays a fundamental role from the acoustic viewpoint, many models for the evaluation of scattering by individual fish, exclusively consider only swimbladder as a non-penetrable body. However for non-swimbladdered fish the fish body modeled as fluid filled geometry requires the solution of a more complicated transmission problem.

2. THE EXCERPT FROM THEORY OF ACOUSTIC SCATTERING

The target strength of an object is the logarithmic acoustic measure of its size and could be defined as:

$$TS = 20log\frac{p_{scat}(r=1,\theta=\pi)}{p_{inc}}[dB] \tag{1}$$

where p_{inc} and p_{scat} are incident and scattered pressure amplitudes. The scattered pressure is referenced to the normalized distance $r_1 = 1$ meter and its value at backscattering direction ($\theta = \pi$) is used in above definition. Numerical modeling of the target strength value requires considering acoustic scattering phenomena with time-harmonic plane wave incident field. When operated at frequency f_0 the normalized incident pressure could be modelled as $p_{inc} = e^{jk_0(d\cdot x)}$ with $k_0 = 2\pi f_0/c_0$ representing wave number, d its direction and c_0 - sound velocity in medium. The acoustic wave are governed by the Helmholtz equation valid in propagated medium Ω_0 with boundary condition defined on surface Γ [4]:

$$\begin{cases} \nabla^2 p_{tot} + k_0^2 p_{tot} = 0 & \text{in} \quad \Omega_0 \\ \alpha p_{tot} + \beta \partial_n p_{tot} = 0 & \text{on} \quad \Gamma \end{cases} \tag{2}$$

where $p_{tot} = p_{scat} + p_{inc}$ is the total pressure and α, β are coefficients depending on object properties. Moreover, the scattered component satisfies Sommerfeld radiation condition $\lim_{r\to\infty} r(\partial_r p_{scat} - jk_0 p_{scat}) = 0$.

The analytical solution of Eq. 2 can be obtained only for several objects with regular geometry including spherical, cylindrical or ellipsoidal ones. This theoretical solutions are often used in asymptotic methods but have also important application when testing software implementations that aims to solve Eq. 2 for any shaped object by the discretisation of its surface.

The Boundary Element Method solutions use variational form of boundary integral equation as the reformulation of Eq. 2 with surface potentials, which uniquely determine the scattered field. The large number of methods of solving integral equations were proposed in the literature and its numerical verification is a topic of last decades contributions not only in acoustics.

As the first example let us consider a problem of soft scattering that has application in modeling swimbladdered fish target strength. This case assumes $p_{tot} = 0$ ($\alpha = 1$, $\beta = 0$) on object surface what gives boundary integral equation and field representation formula in the form:

$$\begin{cases} S \cdot u = -p_{inc} & \text{on} \quad \Gamma \\ p_{scat} = -S_L \cdot u & \text{in} \quad \Omega_0 \end{cases} \tag{3}$$

where u symbolises the surface solution, while S_L and S are single layer potential operator and single layer boundary operator defined by:

$$\begin{cases} S_L = \int_\Gamma G_k(x-y)\psi(y)d\Gamma(y) \\ S = \int_{\Gamma\times\Gamma} G_k(x-y)\psi(x)\phi(y)d\Gamma(x,y) \end{cases} \tag{4}$$

with Green's function $G_k(x-y) = e^{jk|x-y|}/(4\pi|x-y|)$ and discretisation basis functions ϕ and φ. Solution p_{scat} of Eq. 4 could be obtained by assembling boundary operator matrix S, solving linear equation for surface trace u and multiplying it by potential operator S_L.

The second considered case models non-swimbladder fish for which its body reflects the sounding wave. This case requires solving acoustic transmission problem as the internal part of the fish represents another medium Ω_1 in which the wave could propagate with wave number $k_1 = 2\pi f_0/c_1$. The solution could be represented by single-trace formulation following [5]:

$$\begin{cases} \left[\mathbf{A_0} + \widehat{\mathbf{A_1}}\right] \cdot \begin{bmatrix} u_0 \\ u_1 \end{bmatrix} = \begin{bmatrix} -p_{inc} \\ -\partial_n p_{inc} \end{bmatrix} & \text{on} \quad \Gamma \\ p_{scat} = D_L \cdot u_0 - S_L \cdot u_1 & \text{in} \quad \Omega_0 \end{cases} \tag{5}$$

where $\mathbf{A_0} = \begin{bmatrix} -D_0 & S_0 \\ H_0 & T_0 \end{bmatrix}$ represents Calderon projector in surrounding medium Ω_0 with four assembled matrices from single S, double D, adjoined double T and hypersingular H boundary operators, whereas $\widehat{\mathbf{A_1}} = \begin{bmatrix} -D_1 & \frac{\rho_1}{\rho_0} S_1 \\ \frac{\rho_0}{\rho_1} H_1 & T_1 \end{bmatrix}$ is its scaled version for part in internal medium Ω_1. Reconstruction of scattered field p_{scat} uses introduced earlier single layer potential operator and double layer one:

$$D_L = \int_\Gamma \partial_y G_k(x-y)\psi(y)d\Gamma(y) \tag{6}$$

It is worth to note that from mathematical point of view the first considered case represents the spacial case of the second for which $c_1 \to 0$, $\rho_1 \to 0$ and then block matrix Eq. 5 reduces to ordinary matrix Eq. 3 with one single layer potential operator matrix. This fact allows using Eq. 5 for both presented cases. However, as the Calderon projector contains hypersingular operator the solution procedure for the second case requires applying dedicated matrix assembling and regularisation techniques.

3. SOFTWARE IMPLEMENTATION AND RESULTS

The presented boundary element method equations as applied to meshed objects were implemented with the usage FreeFEM and Bempp software packages. Both were used for implementing the the model of swimbladdered fish scattering pattern, whereas only Bempp was used to model non-swimbladdered fish pattern.

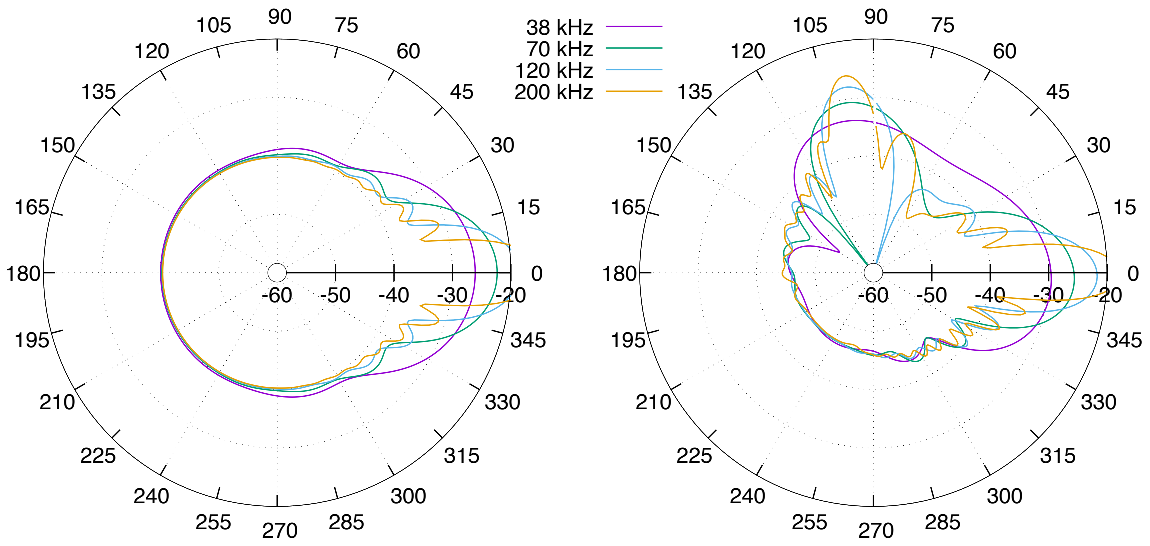

Figure 1: The scattering patterns for soft sphere having diameter equal to 38.1mm (left) and mesh model of 40cm yellowfin tuna swimbladder (right) calculated for sounding frequencies used in fishery acoustics. Backscattering direction is at $\theta = 180°$.

Implementation of soft scattering code was verified by using meshed sphere having radius of 1.905cm with variable number of elements, which in fact imitates spheres used for calibrating underwater equipment. As expected increasing number of elements increases accuracy as compared to theoretical calculations. For both packages the number of source code lines were around 30. However, the FreeFEM code is more difficult in reading as it must define type of variables, what in such specialised problem is not easy to master. The test of speed execution was performed for discretized swimbladder of 40cm yellowfin tuna [6] having mesh with 7502 nodes on M1 MacBook Pro running Sequoia 15.3.2 operating system. In this case FreeFEM v4.14 outperforms Bempp v0.4.2 in calculating scattering pattern having 360 points. The FreeFEM executable runs in 111s as compared to 188s for Bempp_cl implementation.

Fig. 1 presents two figures with patterns prepared for four operated frequencies typically used in fishery acoustics, namely 38kHz, 70kHz, 120kHz and 200kHz. The left pattern presents the results obtained for the model of a sphere with diameter equal to 38.1mm, wheres the right one for a model of 40cm yellowfin tuna.

The second case of fluid-filled targets was implemented in Bempp as it implements operators for transmission problem. Fig. 2 presents 40 lines of Python code that may serve as the minimum working example for solving fish scattering problems using Bempp. To be readable, it is neither parameterised nor modular. It is worth to point out that: (1) the input mesh should be prepared in Gmsh mesh format and should be prepared with resolution appropriate to operating frequency (2) it is assumed that the incident plane wave is arriving from y-axis coordinate (3) the observation circle is defined in x-y plane and contain 1 degree separated points (4) the results of calculation are output to console in the form of two-column table with the angle and absolute value of scattered pressure. The code was executed for 40cm modeled mackerel body having 8582 nodes and its execution in the same scenario as mentioned previously took 17 minutes.

Fig. 3 presents the input meshes for two test cases representing swimbladder of yellowfin tuna and the body of non-swimbladdered mackerel.

```python
# MM 21.5.2025: plane wave scattering from elastic body
import bempp_cl.api
from bempp_cl.api.operators.boundary import helmholtz as boundary
from bempp_cl.api.operators.potential import helmholtz as potential
from numpy import pi, sin, cos, exp, abs, array, concatenate
from scipy.sparse.linalg import gmres

# input parameters
f0, N = 38e3, 360                  # frequency, number of output points
c0, rho0, c1, rho1 = 1480, 1024, 1540, 1045 # sea water - fluid
k0, k1 = 2*pi*f0/c0, 2*pi*f0/c1             # wave numbers
d = array([0, -1.0, 0])                     # plane wave direction
r1 = array([[sin(2*pi*i/N),-cos(2*pi*i/N),0] for i in range(N)]).T

# target mesh and its space
grid = bempp_cl.api.import_grid("macXray_carn_L40cm.msh")
Sp = bempp_cl.api.function_space(grid, "P", 1)

# operators
A0 = boundary.multitrace_operator(grid, k0) # external Calderon
A1 = boundary.multitrace_operator(grid, k1) # internal Calderon
SL = potential.single_layer(Sp, r1, k0)
DL = potential.double_layer(Sp, r1, k0)

# incident pressure and its normal derivative
@bempp_cl.api.complex_callable
def pinc(x, n, idx, res): res[0] = exp(1j*k0*d.dot(x))
f = bempp_cl.api.GridFunction(Sp, fun=pinc).coefficients
@bempp_cl.api.complex_callable
def dpinc(x, n, idx, res): res[0] = 1j*k0*exp(1j*k0*d.dot(x))*d.dot(n)
g = bempp_cl.api.GridFunction(Sp, fun=dpinc).coefficients

# solution
A1[0,1] *= rho1/rho0; A1[1,0] *= rho0/rho1  # scaling Calderon
A = (A0 + A1).strong_form()
u,_ = gmres(A * A, A * concatenate([f,g]))  # surface solution
u1 = bempp_cl.api.GridFunction(Sp, coefficients=u[: len(f)])
u2 = bempp_cl.api.GridFunction(Sp, coefficients=u[len(f) :])
pscat = DL * u1 - SL * u2                   # far field solution
print("\n".join(["%d\t%g"%(i,abs(pscat[0][i])) for i in range(N)]))
```

Figure 2: Minimum working example of Bempp code for the calculation of a scattering pattern of meshed object

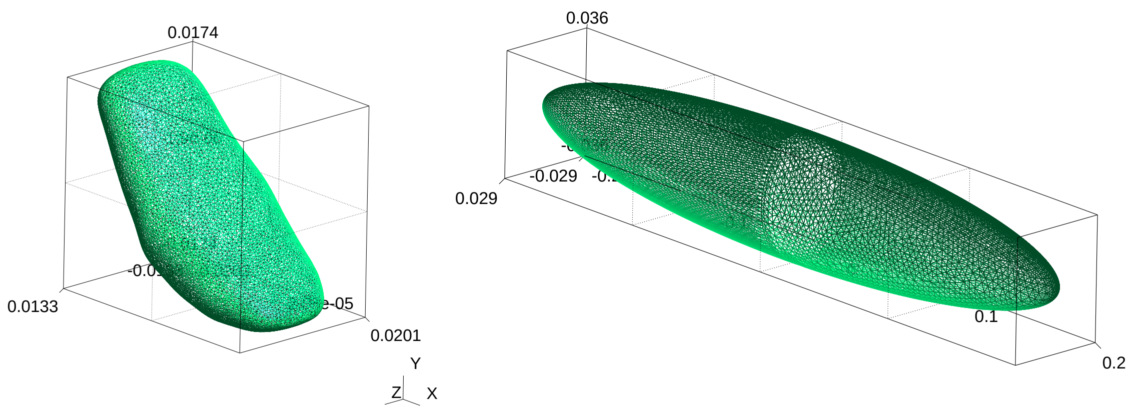

Figure 3: 40cm yellowfin tuna swimbladder mesh model (left) and 40cm mackerel body mesh model (right)

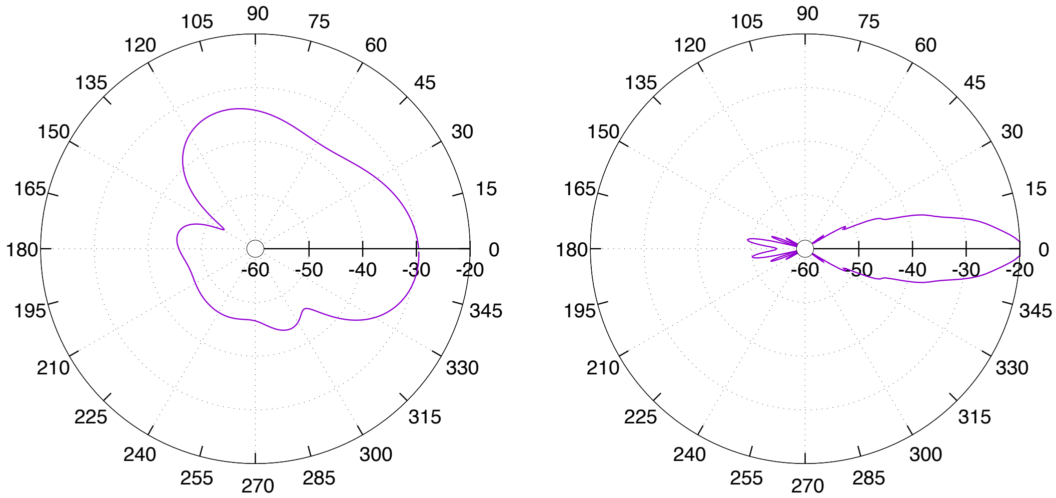

Figure 4: 40cm yellowfin tuna swimbladder scattering pattern (left) and 40cm mackerel body scattering pattern (right). Backscattering direction is at $\theta = 180°$

4. CONCLUSIONS

The paper aimed to verify usefulness of open-source finite and boundary element packages for applications in modeling of fish scattering properties. The two software packages differs by entry level as FreeFEM uses its own language and is dedicated rather for FEM approach. Bempp although is not as fast as FreeFEM allows for clear coding of complicated transmission problems as required for modeling scattering of fish without swimbladder. The execution of this code is far more slower than soft-scattering one.

REFERENCES

[1] Timo Betcke and Matthew W. Scroggs. Bempp-cl: A fast python based just-in-time compiling boundary element library. *Journal of Open Source Software*, 6(59), 2021.

[2] J.D. Gonzalez, E.F. Lavia, S. Blanc, M. Maas, and A. Madirolas. Boundary element method to analyze acoustic scattering from a coupled swimbladder-fish body configuration. *Journal of Sound and Vibration*, 486:115609, 2020.

[3] F. Hecht. New development in freefem++. *J. Numer. Math.*, 20(3-4):251–265, 2012.

[4] Stephen Martin Kirkup. The boundary element method in acoustics: A survey. *Journal of Applied Sciences*, 2019.

[5] Elwin van 't Wout. The boundary element method for acoustic transmission with nonconforming grids. *Journal of Computational and Applied Mathematics*, 445:115838, August 2024.

[6] Anderson Ladino Velásquez. *Acoustic Techniques for Tuna Biomass Estimation*. PhD dissertation, Institut per a la Gestió Integrada de Zones Costaneres, Universitat Politècnica de València, 2024.