

# A BEM model for underwater acoustic radiation due to vibrating vehicle in the sea waveguide

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**Abstract:** *Underwater acoustic radiation behaviour of a flexible sea vehicle in finite water depth is investigated and numerical predictions are presented. To calculate the acoustic radiation field from vibrating ships and marine structures in the sea-acoustic waveguide representations are developed based on modal expansion of the solution of the Helmholtz equation, satisfying the free surface and the seabed boundary conditions. The eigenmodes and eigenfrequencies of the marine structure used in the above representation are calculated by finite element (FEM) solutions. Effects of the free surface and finite water depth are introduced by using a 3D Boundary Element Method (BEM) model, based on the ocean waveguide Green's function obtained by the multiple image-source method. The present model aims to the development and optimization of a computational model for vibro-acoustic calculation associated with the underwater noise radiated from ship and underwater vehicles (URN). Along with measurements, accurate URN modelling will enable the design of quieter vessels, thus contributing to the compliance with the threshold values for continuous noise developed under the European Commission's Marine Strategy Framework Directive (MSFD).*

**Keywords:** *Underwater noise radiation, structural vibration and noise, ocean acoustic waveguide, 3D-BEM modelling.*

## 1. INTRODUCTION

Reducing Underwater Radiated Noise (URN) generated by waterborne transport (shipping) is crucial to mitigate its impact on marine ecosystems. IMO [1] has issued guidelines aiming at the reduction of underwater noise from commercial shipping to address adverse impacts on marine life. Noise is generated by three ship components (machinery, propeller, hull) and the associated URN effects are mainly due to engine noise and vibration, propeller cavitation and fluid hull structure interaction and vibration [2].

Machinery noise excites vibrations of the hull structure resulting in radiation of low frequency noise into the sea environment. Underwater acoustic radiation induced by structural vibration and propagation in the ocean environment involves analyzing the interaction between the acoustic field and the structure. In this work, a hybrid scheme is implemented, integrating the Finite Element Method (FEM) [3] for the determination of structural modes of vibration, with the Boundary Element Method (BEM) for the scattering and propagation of vibrational noise into the sea acoustic waveguide [4].

First, the hydroelastic problem is formulated in the frequency domain for the underwater structure of a flexible vessel in the absence of external forces and damping. The eigenmodes and eigenfrequencies of the structure are calculated using FEM analysis. Then, the hydroacoustic problem concerning the acoustic radiation from vibrating marine structures in the sea acoustic waveguide is formulated using the appropriate Green's function, obtained by the multiple image-source method, along with the satisfaction of the free surface and the seabed boundary conditions. The 3D scattering problem is solved using a BEM model forced by boundary data calculated by the eigenmodes of the hydroelastic problem through the boundary condition on the vibrating hull surface. Results of the predicted acoustic field are presented for two vessels, a travelling ship and an Autonomous Underwater Vehicle (AUV).

## 2. PREDICTION HYDROELASTIC-HYDROACOUSTIC MODELS

### 2.1 The hydroelastic model

The equation of motion for the underwater structure is the basis of the hydroelastic model:

$$\mathbf{M}\mathbf{U}''(t) + \mathbf{C}\mathbf{U}'(t) + \mathbf{K}\mathbf{U}(t) = \mathbf{F}_a(t) + \mathbf{F}_b(t), \quad (1)$$

in which  $t$  is the time instant;  $\mathbf{M}$ ,  $\mathbf{C}$  and  $\mathbf{K}$  denote the mass matrix, damping matrix and stiffness matrix;  $\mathbf{U}''(t)$ ,  $\mathbf{U}'(t)$  and  $\mathbf{U}(t)$  represent the acceleration, velocity and displacement vector;  $\mathbf{F}_a(t)$ ,  $\mathbf{F}_b(t)$  are the mechanical force vector and the hydrodynamic force vector respectively. Assuming time-harmonic vibrations, the following representation is used:

$$\mathbf{U}(t) = \text{Re}\{\mathbf{U} \exp(i\omega t)\}, \quad \mathbf{F}_a(t) = \text{Re}\{\mathbf{F}_a \exp(i\omega t)\}, \quad \mathbf{F}_b(t) = \text{Re}\{\mathbf{F}_b \exp(i\omega t)\}, \quad (2)$$

where  $\omega$  is the angular frequency,  $i^2 = -1$ ,  $\text{Re}\{\cdot\}$  denotes the real part of a quantity, and  $U$ ,  $F_a$  and  $F_b$  represent the displacement vector, mechanical force vector and hydrodynamic force vector in the frequency domain. Therefore, Eq. (1)

$$-\omega^2 \mathbf{M}U + i\omega \mathbf{C}U + \mathbf{K}U = F_a + F_b . \quad (3)$$

In the absence of external forces and neglecting damping effects, Eq.(3) is simplified to the governing equation of the free vibration analysis of a linear elastic structure:

$$-\omega^2 \mathbf{M}U + \mathbf{K}U = 0 , \quad (4)$$

where  $U$  can be expressed by the modal functions as

$$U = \sum u_j c_j , \quad (5)$$

with  $u_j$  and  $c_j$  ( $j=1, 2, \dots$ ) denoting the eigenmodes and the corresponding amplitudes respectively. By utilizing the modal superposition method, the corresponding eigenvalue problem can be formed as

$$(-\omega_j^2 \mathbf{M} + \mathbf{K})u_j = 0 , \quad j = 1, 2, 3, \dots , \quad (6)$$

where  $u_j$  is the  $j$ -th eigenmode and  $\omega_j$  is the  $j$ -th eigenfrequency. It should be noted that the eigenmode corresponds to the mode shape of structural vibration, and the eigenfrequency corresponds to the natural frequency of structural vibration.

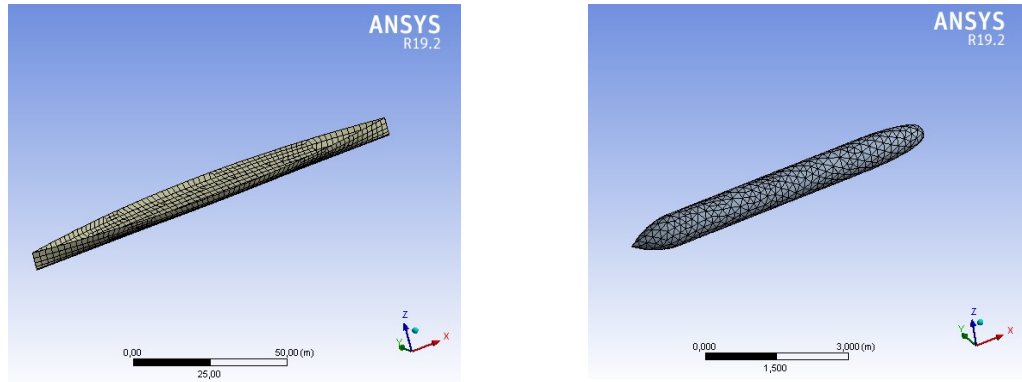


Fig. 1: Discretized vessels' surface using finite elements. Left: Surface ship. Right: AUV

The eigenvalue problem is solved using a FEM modal analysis implemented in the ANSYS Mechanical software [5]. Two vessels, considered as hollow flexible structures with vibrating surface are considered, a travelling ship and a submerged AUV. The ship hull is modelled using the parabolic Wigley hull geometry, with length and breadth chosen as  $L = 130 \text{ m}$  and  $B = 25 \text{ m}$ , respectively, and the draft is set to  $T = 6 \text{ m}$ . The AUV geometry is modelled using the typical Myring geometry [6], with  $L = 7 \text{ m}$ ,  $a = 1.5 \text{ m}$ ,  $b = 1 \text{ m}$ ,  $d = 0.7 \text{ m}$  and  $n = 3$ . In both cases the material of the hull is considered structural steel with density  $\rho = 7850 \text{ kg/m}^3$  and Young's modulus  $E = 2 \cdot 10^{11} \text{ Pa}$ . The thickness of the ship surface is set to  $1.6 \text{ cm}$ , whilst the thickness of the AUV surface is set to  $5 \text{ cm}$ . For

the application of FEM, the ship surface is discretized with 900 elements, whilst the AUV surface is discretized with 3000 elements (Fig. 1).

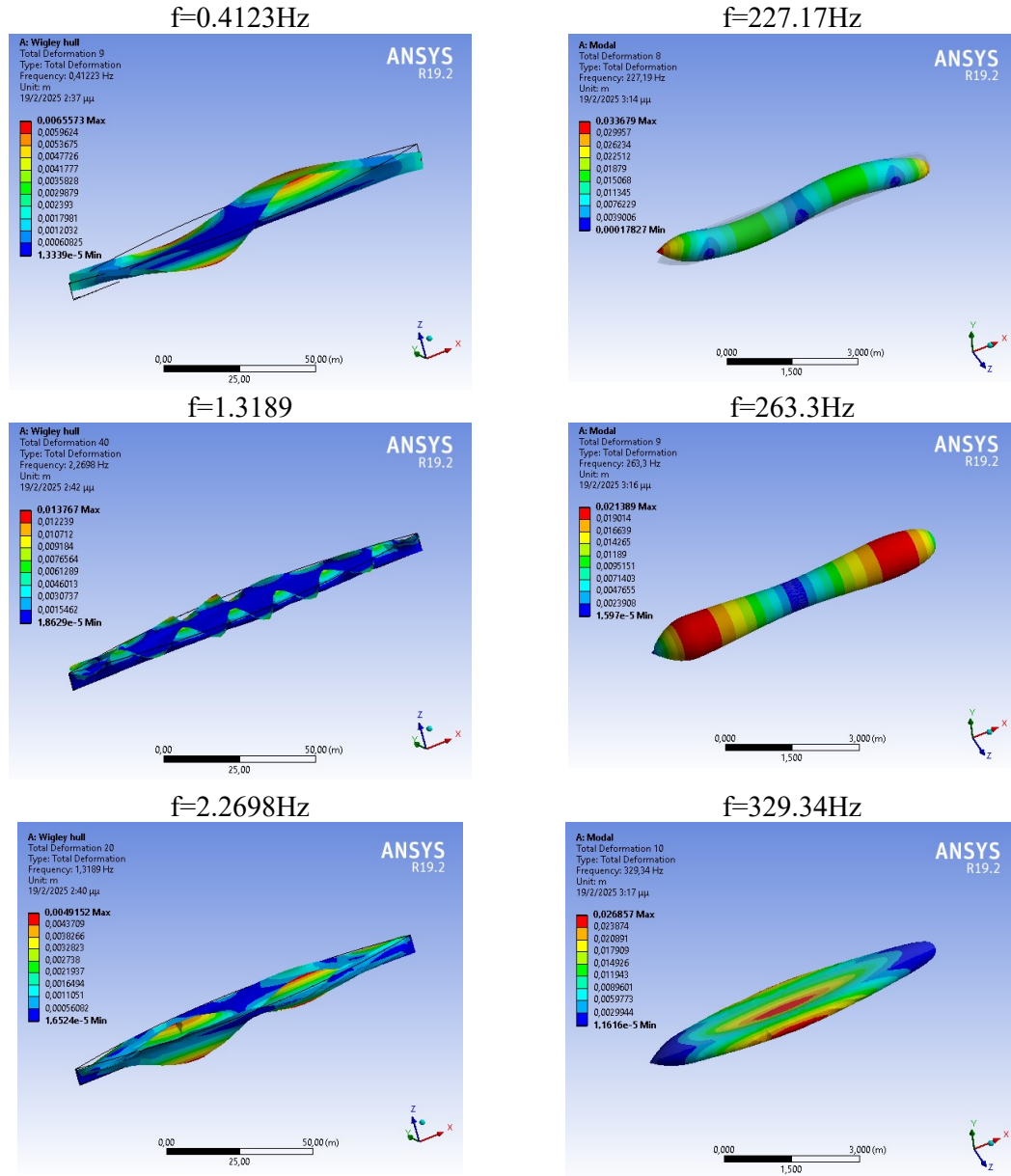


Fig. 2: Total deformations at different eigenfrequencies. Left: ship hull, Right: AUV

The deformations of the ship surface for selected eigenfrequencies are presented in Fig. 2 (left). They exhibit a wavy pattern with a characteristic length inversely proportional to the eigenfrequency, mainly in the direction of the long dimension of the ship. Note that the absolute value of deformations is not important since they correspond to the free response of the structure obtained by modal analysis. Due to the much smaller dimensions the eigen modes of the studied AUV are significantly larger. Deformations for selected eigenmodes are presented in Fig. 2 (right). Due to the closed surface and the smaller size, the deformation pattern is much different compared to the ship. It is interesting to observe that for the higher eigenfrequencies of 263 and 329  $\text{Hz}$  deformations occur along the lateral directions  $y$  and  $z$ .

## 2.2 The hydroacoustic model in the frequency domain

In this part the hydroacoustic model is formulated to predict underwater acoustic radiation and propagation resulting from structural vibration in the ocean acoustic waveguide at specific eigenfrequencies. According to Eq.(5) the acoustic response for any excitation is obtained by superposition of the acoustic fields associated with the vibration modes, therefore the following approach provides the general treatment of the problem.

The acoustic waveguide consists solely of seawater, limited above by the free surface and below by the fully reflective, hard seabed surface, respectively. In the frequency domain the acoustic wave velocity potential  $\varphi$  induced by the structural vibrations will satisfy the Helmholtz equation. According to the modal superposition method, each mode  $\varphi_j$  of the velocity potential corresponding to the  $\omega_j$  eigenfrequency of structural vibrations will also satisfy the Helmholtz equation, and the same applies for each mode of the acoustic pressure  $p_j$ . Therefore,

$$\Delta p_j + k_j^2 p_j = 0, \quad (7)$$

where  $\Delta$  denotes the Laplacian and  $k_j$  represents the wave number that corresponds to the  $\omega_j$  eigenfrequency.

Given the displacements  $u_j(\mathbf{x})$ , as provided by the solution of the eigenvalue problem (6), the scattering problem is considered, excited by the structural vibrations on the wetted surface of a surface ship or AUV vessel, also accounting for the scattering effects of the free-surface and seabed boundary of the ocean acoustic waveguide. The mathematical description of the scattering of time-harmonic waves by an obstacle  $D$  leads to a Boundary Value Problem (BVP) for the Helmholtz equation. Considering the boundary of the obstacle  $\partial D$  to be acoustically hard the following simplified boundary condition is imposed on the wetted surface  $\partial D$  of the body:

$$\frac{\partial p_j(\mathbf{x})}{\partial n(\mathbf{x})} = i\omega_j u_j(\mathbf{x}) = g_j(\mathbf{x}), \quad j=1,2,3,\dots, \quad \mathbf{x} \in \partial D, \quad (8)$$

indicating that the normal displacement at the interface between the structure and the sound field is continuous. In the above equation  $\mathbf{n}(\mathbf{x})$  is the unit vector normal to  $\partial D$  with a direction towards the exterior of  $D$ .

In the case of the acoustic waveguide,  $p_j(\mathbf{x})$  should also satisfy the homogeneous Dirichlet BC,  $p_j = 0$  at the sea surface,  $z = 0$ , and the Neumann BC,  $\partial p_j / \partial n = 0$  at the hard sea bottom,  $z = -h$ . The solution of the scattering problem is given by the integral

$$p_j(\mathbf{x}) = \int_{\partial D} G(\mathbf{x}, \mathbf{y}) \sigma_j(\mathbf{y}) dS(\mathbf{y}), \quad j=1,2,3,\dots \quad \mathbf{x} \in R^3 \setminus \partial D, \quad (9)$$

where  $G(\mathbf{x}, \mathbf{y})$  is the Green's function of the Helmholtz equation in the waveguide, given by either by a normal series expansion [7], or by a superposition of fields of monopole sources according to the multiple image model. The distribution of source intensities on

the body surface,  $\sigma_j(y)$ , is provided as the solution of the BVP, discretized and solved using a 3D BEM. More details can be found in [8].

The underwater sound pressure  $p$  in the ocean environment is approximated by the summation of the pressure contributions associated with each structural mode:

$$p = i\omega\rho_\alpha \sum p_j c_j, \quad (10)$$

where  $\rho_\alpha$  is the water density, and the amplitudes  $c_j$  can be calculated to represent the structure response for any load. The sound pressure level  $SPL$  can be calculated using the modulus of the complex sound pressure  $|p|$ ,  $SPL = 20 \log(|p|/p_{ref})$ ,  $p_{ref}$  is the reference sound pressure.

### 3. NUMERICAL RESULTS AND DISCUSSION

The developed BEM is applied to the solution of the scattering problem and the prediction of the acoustic field in the case of noise emission from the vibrations of the two flexible vessels, for which a modal analysis has been performed as presented in section 2.1. The displacements calculated by the modal analysis are used in the boundary condition (8) on the wetted surface of the vessel. In both cases, an isovelocity environment  $c = 1500 \text{ m/s}$  modelling an ocean acoustic waveguide of depth  $h = 500 \text{ m}$  is considered. For the travelling ship the submergence depth is taken equal to  $4.5 \text{ m}$ . The selected Froude number corresponds to a vibrational frequency of  $4.1567 \text{ Hz}$ . A number of  $25 \times 10 = 250$  elements are proven to be sufficient for the discretization of the hull.

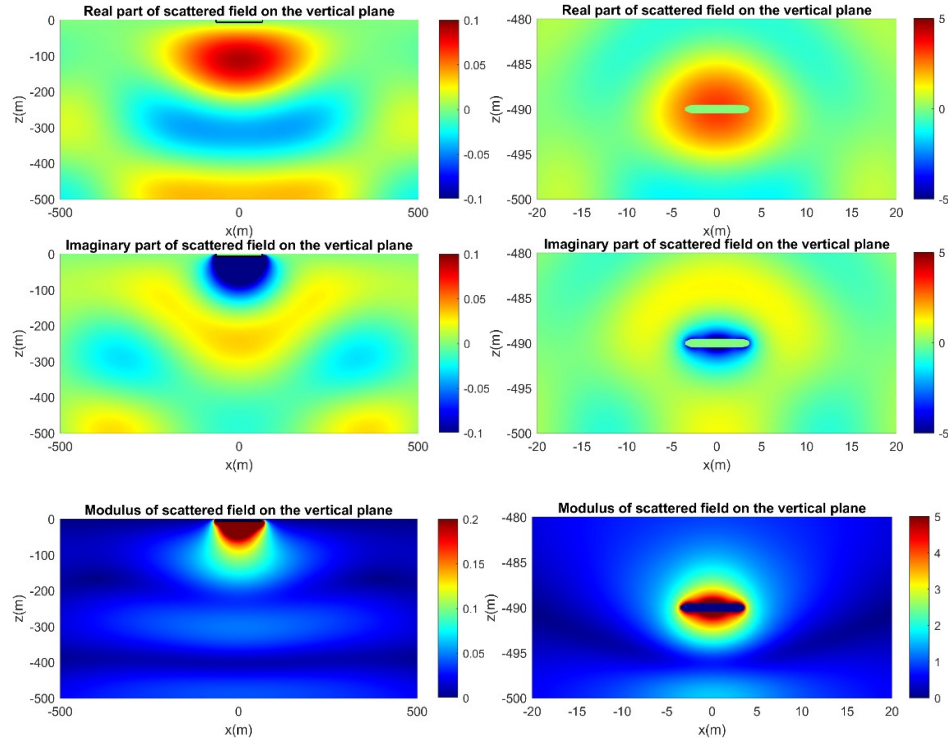


Fig. 3: Plots of acoustic field on a vertical plane passing through the major hull axis. Left: Surface ship at  $f=4.1567 \text{ Hz}$ . Right: AUV at  $f=91.963 \text{ Hz}$ .

The submergence depth of the AUV is taken as  $490m$ ,  $10m$  above the seabed. Compared to the ship, the size of the vessel is significantly smaller, and its submergence depth is large, therefore focus is made on a smaller region around the AUV hull, close to the seabed. A number of  $25 \times 15 = 375$  elements are used for the discretization of the hull. The selected Froude number corresponds to a vibrational frequency of  $91.963Hz$ . In Fig. 3, the contours of the real and imaginary part, as well as the modulus of the acoustic field are presented on a vertical  $xz$ -plane passing through the major axis of the hull. The presence of both the seabed and the free surface significantly affects the directionality of the acoustic field. The pattern of the acoustic field in the  $z$ -direction is governed by the characteristic wavelength. In the case of AUV, due to the submergence of the whole vessel, noise is emitted to all directions. In the plot of the modulus the mirror effect caused by the presence of the seabed is pronounced, whilst, as expected, the effect of the free surface has weakened in the submergence depth of  $490m$ .

#### 4. CONCLUSIONS

In the present work, the acoustic radiation from the vibration of flexible sea vehicles into the ocean acoustic waveguide is studied. The 3D scattering problem is treated by developing a 3D-BEM model in the frequency domain. Deformations of the wetted surface of the marine vehicles is introduced through the boundary condition indicating that the normal displacement at the interface between the structure and the sound field is continuous. The effects of the free surface and the seabed are considered through the corresponding boundary conditions of the 3D scattering problem. In the ship case, results show that the effects of the hull scattering, the seabed and the sea surface are important for the directionality of the generated noise. In the AUV case, due to the larger submergence depth, the effect of the free surface is weak, and the directionality is mainly affected by the body scattering and the presence of the seabed. This result may be further exploited to consider the response of the ship and AUV vibrations at certain excitations from the machinery noise which is one of the main components of noise generated from sea vehicles. The present model will contribute to the development of a computational model for URN modelling, aiming to the design of quieter vessels, in compliance with the threshold values for continuous noise developed under the European Commission's Marine Strategy Framework Directive (MSFD).

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