# Distance estimation of near-surface moving targets by deep sonar

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***Abstract:*** *Using deep sonar for near-surface target positioning, an inaccurate sound speed profile can result in significant deviation in horizontal distance estimation. Based on a few model assumptions, we propose a high-precision horizontal distance estimation polynomial that eliminates the need for knowledge of the sound speed profile. The input parameters to the polynomial are the echo time delay and incident angle. The polynomial coefficients are only related to the target depth and the sound speed profile. If the depth of the moving target remains constant, the polynomial coefficients can be obtained by estimating the time delays and incident angles of multiple echoes from the moving target. The target's horizontal distance can be obtained by inputting the coefficients, echo time delay and incident angle into the polynomial. Under the MUNK standard sound speed profile, simulation results verify that the proposed method's horizontal distance estimation deviation does not exceed 20 meters. This method is suitable for the design of deep-sea navigation and detection systems.*

***Keywords:*** *Deep sea, Large receiving depth, Target distance estimation.*

## 1. INTRODUCTION

In the deep sea, deploying sonar near the seabed to detect near-surface targets presents advantages such as stable acoustic channels, minimal propagation loss, and the absence of acoustic shadow zones for near-surface targets within medium detection ranges, as the direct sound paths of target echoes do not contact the sea surface or seabed. The detection framework has great prospects for application. In the deep sea, the underwater sound speed varies vertically within a certain range, and the curve that describes the variation of sound speed with depth in the deep sea is referred to as the sound speed profile[1]. According to ray theory, due to the variation of sound speed with depth in the deep sea, sound rays are curved. Consequently, underwater target positioning methods based on the assumption of straight sound rays will result in significant deviations. Traditional passive[2, 3, 4, 5] and active[6, 7] positioning methods generally require precise sound speed profile parameters, which is unfavorable for acoustic positioning in unfamiliar maritime areas. In the deep sea, this paper proposes a target distance estimation method that does not require sound speed profile parameters for the scenario of deploying the monostatic sonar near the seabed to actively detect underwater targets.

## 2. DEEP-SEA UNDERWATER SOUND PROPAGATION MODEL

### 2.1. DEEP SEA SOUND SPEED PROFILE

In the deep sea, the underwater sound speed at the same depth is generally the same over a larger area. Except for the sound speed near the sea surface, which is greatly influenced by the ocean environment, the sound speed initially decreases then increases with increasing depth. A typical model for deep-sea sound speed profiles is the Munk sound speed standard model[8]. The Munk sound speed standard model described as

$$c(z) = c_{\text{min}}[1 + \varepsilon(\eta + e^{-\eta} - 1)]. \tag{1}$$

where $\eta = \frac{2(z-z_0)}{B}$, $z$ is depth. Typical data given by Munk is $B = 1000$ m, $z_0 = 1000$ m, $c_{\text{min}} = 1500$ m/s, $\varepsilon = 0.57 \times 10^{-2}$.

### 2.2. DEEP-SEA NEAR-SEAFLOOR SONAR BOTTOM-UP DETECTION MODEL

In the deep sea, when the frequency of sound waves is greater than 500 Hz, it is suitable to describe underwater sound propagation with the ray model[9]. the schematic diagram of a monostatic sonar detecting underwater targets near the seabed is shown in Fig. 1, where $z_N$ and $z_T$ are monostatic sonar depth and target depth, respectively. $R_T$ is the horizontal distance to the target from the sonar. Because the sound speed profile $c(z)$ only varies with depth, the direct sound ray to the target is isotropic in the horizontal direction, and the sonar, target, and sound ray lie within the same vertical plane.

According to [7] and [10], the echo parameters of the target and the parameters of the sound

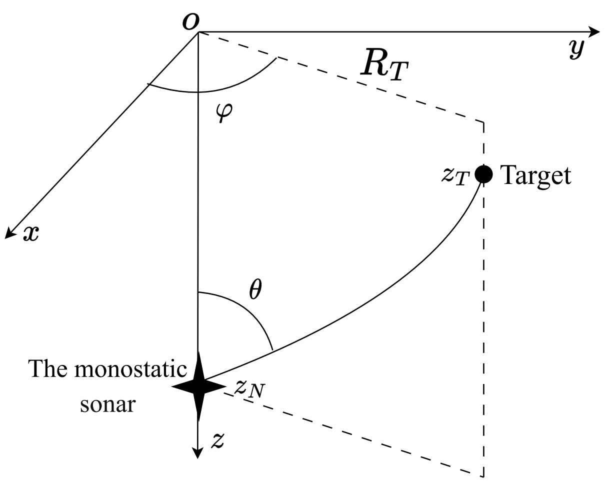



*Figure 1: Deep-sea near-seafloor sonar bottom-up detection model.*

speed profile satisfy the following three equations:

$$\frac{\tau_r}{2} = \frac{1}{c(z_N)} \int_{z_T}^{z_N} \frac{[c(z_N)/c(z)]^2}{\sqrt{[c(z_N)/c(z)]^2 - \sin^2(\theta_r)}} \mathrm{d}z, \tag{2}$$

$$\frac{\tau_r}{2} = \frac{1}{c(z_N)} \left[ R_T \sin(\theta_r) + \int_{z_T}^{z_N} \sqrt{[c(z_N)/c(z)]^2 - \sin^2[\theta_r]} \mathrm{d}z \right], \tag{3}$$

$$R_T = \int_{z_T}^{z_N} \frac{\sin(\theta_r)}{\sqrt{[c(z_N)/c(z)]^2 - \sin^2(\theta_r)}} \mathrm{d}z, \tag{4}$$

where $\tau_r$ is the two-way delay of the target echo, $\theta_r$ is the target's incident angle, where the incident angle refers to the angle between the direction of the echo and the perpendicular line.

## 3. DISTANCE ESTIMATION METHOD FOR A MOVING TARGET WITH CONSTANT DEPTH

According to the Taylor expansion, Eq. 2 can be denoted as

$$\tau_r = \sum_{n=0}^{\infty} \frac{1}{\cos^{2n+1}(\theta_r)} a_n(z_T), \tag{5}$$

where

$$a_0(z_T) = \int_{z_T}^{z_N} \frac{2c(z_N)\mathrm{d}z}{[c(z)]^2}, a_n(z_T) = \frac{(-1)^n(2n-1)!!}{(2n)!!} \int_{z_T}^{z_N} \frac{2c(z_N)x^n(z)\,\mathrm{d}z}{[c(z)]^2}, n = 1, 2, \cdots \infty. \tag{6}$$

Eq. 3 can be denoted as

$$R_T \sin(\theta_r) = \frac{c(z_N)}{2}\tau_r - \sum_{n=0}^{\infty} \frac{1}{\cos^{2n-1}(\theta_r)} b_n(z_T), \tag{7}$$

where

$$b_0(z_T) = \int_{z_T}^{z_N} \mathrm{dz}, b_n(z_T) = \frac{(-1)^{n+1}(2n-3)!!}{(2n)!!} \int_{z_T}^{z_N} x^n(z)\mathrm{d}z, n = 1, 2, \cdots, \infty. \tag{8}$$

Eq. 4 can be denoted as

$$R_T = \tan(\theta_r) \left( \sum_{n=0}^{\infty} \frac{1}{\cos^{2n}(\theta_r)} f_n\left(z_T\right) \right), \tag{9}$$

where

$$f_0\left(z_T\right) = \int_{z_T}^{z_N} \mathrm{dz}, f_n\left(z_T\right) = \frac{(-1)^n(2n-1)!!}{(2n)!!} \int_{z_T}^{z_N} x^n\left(z\right) \mathrm{d}z, n = 1, 2, \cdots, \infty. \tag{10}$$

The convergence condition of the Taylor expansion is

$$\left| \frac{x(z)}{\left[\cos\left(\theta_r\right)\right]^2} \right| < 1, z \in [0, z_N], \tag{11}$$

where $x(z) = \left[c(z_N)/c\left(z\right)\right]^2 - 1$. Since the underwater sound speed generally varies from 1450 m/s to 1540 m/s, $|x(z)| << 1$, $\left[c(z_N)/c(z)\right]^2 \approx 1$, there is the following equation

$$b_n(z_T) \approx \frac{c(z_N)a_n(z_T)}{2\left(1-2n\right)}, f_n\left(z_T\right) \approx \frac{c(z_N)}{2} a_n\left(z_T\right), n = 1, 2, \cdots, \infty. \tag{12}$$

According to Eq. 6, when $n$ is larger, $a_n(z_T)$ approaches zero. Combining Eq. 7, Eq. 9, and Eq. 12, the following equation exists

$$R_T \approx \frac{\frac{c(z_N)}{2}\left[\frac{\tau_r}{\cos(\theta_r)} + \sum_{n=1}^{M-1} \frac{a_n(z_T)}{\cos^{2n}(\theta_r)} \frac{2n}{2n-1}\right]}{\left[\tan\left(\theta_r\right) + \cot(\theta_r)\right]}. \tag{13}$$

According to Eq. 13, $a_n\left(z_T\right)$ is required to estimate $R_T$. The estimation method for $a_n\left(z_T\right)$ is as follows:

According to Eq. 5, if incident angles satisfy Eq. 11, multiple echo parameters $(\theta_{r_i}, \tau_{r_i}), i = 0, 1, ..Q$ for targets at the same depth, but different horizontal position satisfy

$$\tau = \mathbf{A}\mathbf{a}(z_T) + \Delta\mathbf{E}, \tag{14}$$

where $\tau = \begin{bmatrix} \tau_{r_0} & \tau_{r_1} & \cdots & \tau_{r_Q} \end{bmatrix}^{\mathrm{T}}$, $\mathbf{a}(z_T) = \begin{bmatrix} a_0(z_T) & a_1(z_T) & \cdots & a_{M-1}(z_T) \end{bmatrix}^{\mathrm{T}}$, $\Delta\mathbf{E} = \begin{bmatrix} \Delta e_0 & \Delta e_1 & \cdots & \Delta e_Q \end{bmatrix}^{\mathrm{T}}$. and $\mathbf{A} = \begin{bmatrix} \cos^{-1}(\theta_{r0}) & \cos^{-(2n+1)}(\theta_{r0}) & \cdots & \cos^{-(2M-1)}(\theta_{r0}) \\ \cos^{-1}(\theta_{r1}) & \cos^{-(2n+1)}(\theta_{r1}) & \cdots & \cos^{-(2M-1)}(\theta_{r1}) \\ \vdots & \vdots & \ddots & \vdots \\ \cos^{-1}(\theta_{rQ}) & \cos^{-(2n+1)}(\theta_{rQ}) & \cdots & \cos^{-(2M-1)}(\theta_{rQ}) \end{bmatrix}$.

According to the least squares method, $\mathbf{a}(z_T)$ is denoted as

$$\mathbf{a}(z_T) = \left(\mathbf{A}^{\mathrm{T}}\mathbf{A}\right)^{-1} \mathbf{A}^{\mathrm{T}} \tau. \tag{15}$$

We refer to the aforementioned method as the Moving Target Distance Estimation Method(MTDE). If the depth of the moving target remains unchanged, the steps for estimating the target's distance are as follows:

1. Track moving targets and estimate the target's multiple echo parameters $(\theta_{r_i}, \tau_{r_i}), i = 0, 1, ..Q$ at different locations.

2. Construct $\mathbf{A}$ and $\tau$ according to Eq. 14.

3. Calculate $\mathbf{a}(z_T)$ according to Eq. 15.

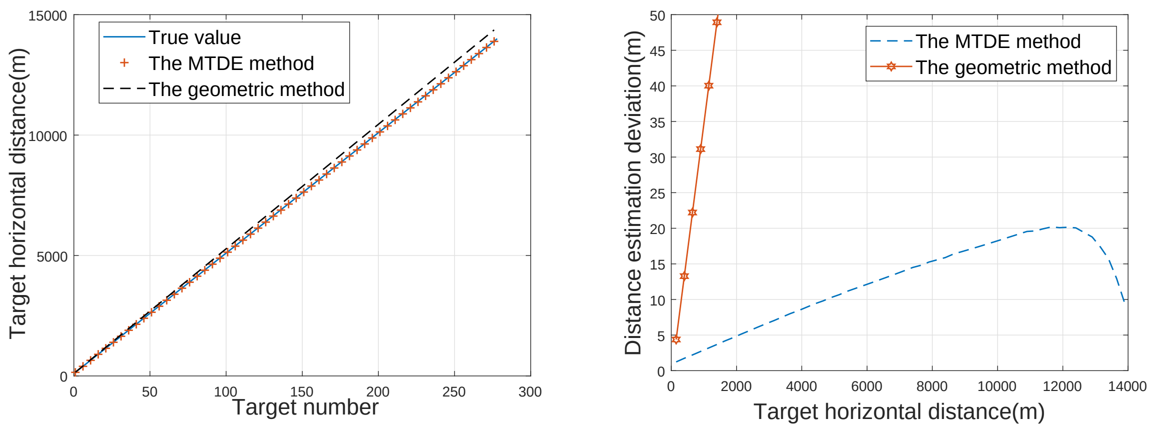
4. Calculate the target's horizontal distance $R_T$ by substituting the target echo parameters $(\theta_r, \tau_r)$ at the current position and $\mathbf{a}(z_T)$ into Eq. 13.

## 4. SIMULATION ANALYSIS

The true value of the target distance is $R_T$, while the estimated value is $\hat{R}_T$. We define the target's distance estimation deviation is $\Delta R = \left|R_T - \hat{R}_T\right|$. Using the Munk standard sound speed profile as a background, the depth of the monostatic sonar is set at 4000 meters underwater. Simulate a moving target with a depth of 100 m approaching sonar from a horizontal distance of 14 km, and calculate the corresponding target echo parameters. What's more, only the sound speed $c(z_N)$ at the depth at which the sonar is located is known when estimating the target distance. The estimated target horizontal distance calculated using a geometric method based on the assumption of a straight-line sound path is

$$\hat{R}_{T_i} = \frac{\tau_{ri} \sin(\theta_{ri})}{2} c(z_N). \tag{16}$$

Select $M$ equals 5, the comparison results among the MTDE method, the geometrical method and the real horizontal distance of the target are shown in Fig. 2.




(a) (b)

*Figure 2: The comparison between the distance estimation and the true value is shown in (a), and the deviation of distance estimation is shown in (b).*

With the Munk standard sound speed profile as the background, the pictures in Fig. 2 show that the estimation results of the MTDE method are more accurate, and the distance estimation deviation is less than 20 m.

## 5. CONCLUSION

In the deep sea, against the background of placing a monostatic sonar system near the seabed for underwater target detection, this paper derives a distance estimation polynomial with the target's incident angle and time delay as independent variables. If the moving target's depth remains constant, this paper introduces a solution for estimating the polynomial coefficients using echo parameters, and further proposes a target distance estimation method that does not require sound speed profile parameters. Under the MUNK standard sound speed profile, simulation results verify that the horizontal distance estimation deviation of the proposed method does not exceed 20 meters.

## 6. ACKNOWLEDGEMENTS

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