FREQUENCY-DIFFERENCE BEAMFORMING IN SHALLOW WATER ENVIRONMENTS

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Abstract: *Conventional beamforming is a signal processing technique that can provide unambiguous ray arrival directions from high signal-to-noise ratio recordings from a transducer array, provided the spacing of the array elements does not exceed half of a signal wavelength. When the array element spacing is significantly larger than a wavelength, spatial aliasing may cause conventional methods to fail. Frequency-difference beamforming [Abadi et al. 2012] was developed to overcome the effects of spatial aliasing by utilizing a surrogate field at a below-band frequency generated from a quadratic product of recorded complex field amplitudes at different frequencies. This below-band frequency (the difference frequency) can be chosen specifically for the needs of the user and is limited only by the frequency resolution and the recorded signal's bandwidth. In this presentation, the performance of frequency-difference beamforming is presented for three shallow ocean scenarios involving realistic multipath propagation. First, the performance of frequency-difference beamforming is presented in a* *simulated ocean channel that is 106 m deep and downward refracting, with a 56 m long, 16 element vertical receiver array placed 3 km away from an 8-element source array. Here, frequency difference beamforming is applied to 11 kHz to 33 kHz frequency sweep signals and compared with conventional beamforming applied to similar signals one decade lower in frequency. Second, frequency-difference beamforming results are presented for vertical array measurements made in a nominally-identical ocean sound channel. Lastly, frequency-difference beamforming is investigated for horizontal arrays in simulated environments with range- and position-dependent sound speeds and depths, and varying receiver position uncertainty. In all three cases, it was found that frequency difference beamforming could be successful when conventional in-band beamforming was not. [Sponsored by ONR and NAVSEA]*

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INTRODUCTION

Conventional (or Bartlett) beamforming methods have been used for decades in determining source location or direction of arrival from an acoustic receiver array output. Many advanced methods have been developed in this time, particularly with the goal of sidelobe suppression and increased resolution [1,2,3,4]. Many sources exist that provide a thorough overview of these methods [5,6,7], so further review is left to the reader.

Common beamforming methods rely on a receiver array geometry that is non-sparse, meaning the spacing between receivers is comparable to the wavelength of the signal of interest. As sparsity increases, i.e. the receiver spacing becomes much larger than the signal wavelength, sidelobes interfere with the actual source beam, distorting the output until the source location can no longer be determined. With few exceptions [8,9], methods correcting for array sparseness do not seem to exist. Recently, frequency-difference beamforming [10] was introduced as a method to beamform with a sparse array for use with blind deconvolution methods. Here, frequency-difference beamforming is demonstrated to be a reliable means of beamforming with a sparse array. First, a brief overview of frequency-difference beamforming is provided. Next, simulations are used to evaluate performance with shallow-water horizontal arrays. Lastly, shallow-water vertical array experiments provide actual ocean measurements to validate against.

BACKGROUND

Conventional beamforming utilizes a simple weighted sum of receiver array outputs to locate a source. The outputs, $p_j(t)$ (time domain) or $P_j(\omega)$ (frequency domain) from the jth receiver element, combined with a weighting vector $w_j(t)$ or $W_j(\omega)$, provide the output $B_{conv}(\omega)$ using

$$B_{conv}(\omega) = \left|\sum_{j=1}^{N} P_j(\omega)W_j^*(\omega)\right|^2 \quad (1)$$

$$W_j(\omega) = exp\{i\omega\hat{\boldsymbol{e}}_s \cdot \boldsymbol{r}_j/c\} \text{ or } W_j^*(\omega) = exp\{i\omega|\boldsymbol{r}_s - \boldsymbol{r}_j|/c\} \quad (2)$$

where ω is the frequency in radians, c is the sound speed, and $\boldsymbol{r}_s$ and $\boldsymbol{r}_j$ are the source and receiver locations, respectively. The choice of $W_j(\omega)$ depends on plane-wave (eq. 2a) or spherical-wave (eq. 2b) beamforming. Often a $1/r$ amplitude weighting is used, but for simplicity, it is not included here.

Frequency-difference beamforming is structured similarly, but rather than using the direct receiver outputs, a function called the "autoproduct" is used. Equation 3 defines the autoproduct for ray-based propagation, which is simply the frequency-domain product between a receiver output at two different frequencies, using the complex conjugate of one of them.

$$AP_{\Delta j} = P_j(\omega_2)P_j^*(\omega_1) = e^{i(\varphi_s(\omega_2)-\varphi_s(\omega_1))}\left\{\sum_{l=1}^{L} A_{lj}^* A_{lj}\, e^{i\left(\frac{\Delta\omega r_{lj}}{c}\right)} + neglected\ terms\right\} \tag{3}$$

Here, A_{lj} is the amplitude of the lth ray path at the receiver. The result is that the autoproduct creates a phase term at a difference frequency, $\Delta\omega = \omega_2 - \omega_1$, which can be seen in more detail in [10] and [11]. One can beamform with the autoproduct at the out-of-band difference frequency, rather than the in-band frequencies used in conventional beamforming.

$$B_\Delta(\omega_1, \omega_2, \Delta\omega) = \left|\sum_{j=1}^{N} P_j(\omega_2)P_j^*(\omega_1)W_j^*(\Delta\omega)\right|^2 \tag{4}$$

By beamforming at the lower, out-of-band difference frequency, beamforming outputs comparable to lower frequency, in-band signals can be achieved. Figure 1 demonstrates this with a simple example of conventional beamforming outputs from a near-field, constant sound speed experiment with 1.5 inch receiver spacing using a 0.20 ms, 10 kHz pulse ($d/\lambda = 0.254$), and a 0.20 ms, 140-180 kHz sweep ($d/\lambda = 4.06$ at 160 kHz). The third panel also used the 140-180 kHz sweep, but with frequency-difference beamforming at $\Delta f = 10\ kHz$. The expected source location is marked with a black circle, and the beamformer peak with either a red or white circle.

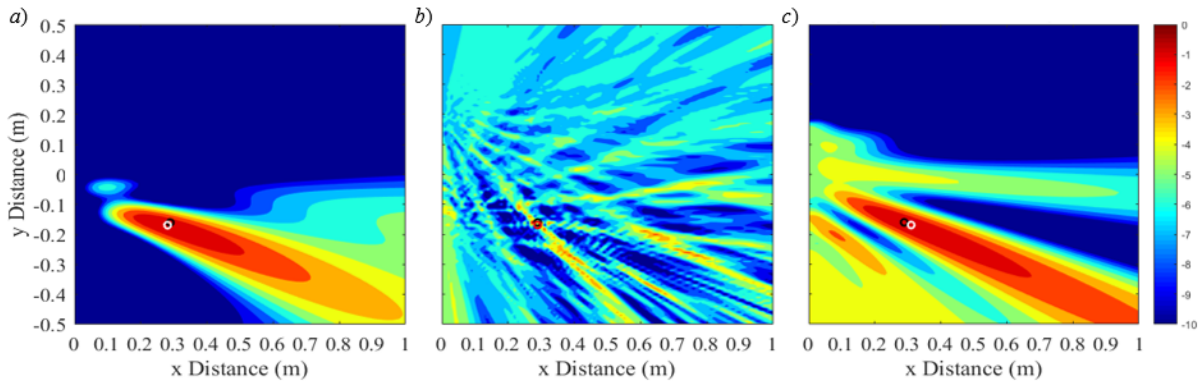

Fig. 1: a) Conventional beamforming on a 10 kHz pulse, b) conventional beamforming on a 140-180 kHz sweep, showing multiple peaks in the window and c) frequency-difference beamforming on a 140-180 kHz sweep with Δf = 10 kHz.

This example clearly illustrates that frequency-difference beamforming in a free-space propagation scenario can produce outputs comparable to conventional beamforming, when the in-band conventional beamforming matches the difference frequency used on a higher frequency signal.

HORIZONTAL ARRAYS IN SHALLOW WATER

Towed horizontal arrays are of practical importance in many applications. In the following simulations, performed using the Bellhop and Kraken Matlab codes (provided by HLS Research), the sound channel geometry and downward-refracting sound speed profile are borrowed from a Kauai Acomms MURI 2011 (KAM11) experiment, shown in Figure 2. However, a single receiver depth is chosen and the receivers placed at varying ranges from the receiver, simulating a horizontal array. The source is kept at 3km for all cases shown here and the results are examined for two geometries: 5 m receiver spacing with a 5° arrival angle, and 50 m receiver spacing with 55° arrival angle. The simulated signals consist of 100 ms, 1-3 kHz frequency sweeps, making the receiver spacing either 6.7 or 67.7 wavelengths apart at the 2 kHz center frequency. The receiver and source depths considered are 50 and 90 m, respectively, chosen for prevalence of multipath propagation.

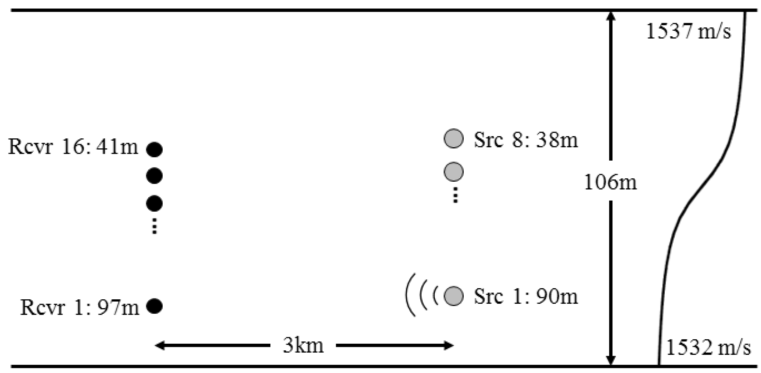

Fig. 2: KAM11 shallow-water geometry. For the horizontal array case, a single source depth and receiver depth are chosen with varying range.

Figure 3 shows the beamforming output from the two cases considered. Each plot compares frequency-difference beamforming at 150 Hz or 15 Hz using the 1-3 kHz bandwidth, conventional beamforming over 1-3 kHz, and conventional beamforming on a 150 or 15 Hz signal (i.e. an in-band frequency that matches the difference frequency). It is clear that frequency-difference beamforming provides a result that will likely behave more robustly than the conventional high-frequency output. Though the conventional output correctly localizes the source, it is evident that this output could deteriorate with added uncertainty due to the lower peak-sidelobe ratio, especially in the 55° case, in which the peak-sidelobe ratio is insignificant. In addition, frequency-difference beamforming provides an output comparable to that of conventional beamforming at an in-band frequency matching the difference frequency, with overlapping sidelobes and comparable peak-sidelobe ratio for the smaller-angle case.

To evaluate the method's robustness, the same cases are considered, but the individual ray paths are varied by applying random delays to account for varying environmental conditions, channel geometry, or receiver array geometry. The delays are determined by choosing a new Gaussian random variable centered around zero for each path. This is repeated at multiple standard deviations for the random variable, a total 100 times at each standard deviation. The outputs for both beamforming methods are compared by considering the error in the location of the beamformer peak for each trial. Figure 4 shows the error for all 100 cases of each standard deviation. These plots make it clear that frequency-difference beamforming significantly outperforms conventional beamforming for the lower-delay values and they both fail with higher delays. In general, it appears that frequency-difference beamforming performs well (i.e. only several "failure" cases) if the product between the average standard deviation of the delays and the difference frequency remains at approximately 0.20 or less.

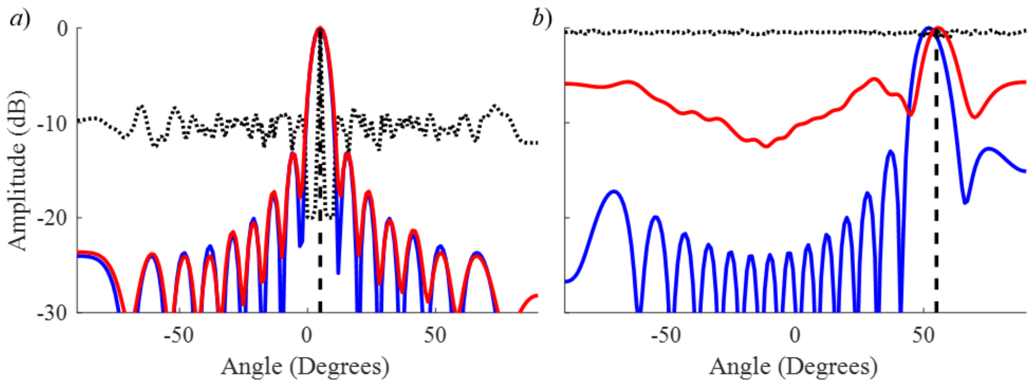

Fig. 3: Comparisons of beamforming outputs for a horizontal array with a) 5m spacing and a source at 5° and b) 50m spacing and a source at 55°. The frequency-difference beamforming (red lines) at 150 (left) and 15 Hz (right) difference frequency, conventional in-band beamforming (dotted black lines), and conventional beamforming (blue lines) using 150 (left) and 15 Hz (right) signals, are all compared with the expected arrival angle (vertical dashed line).

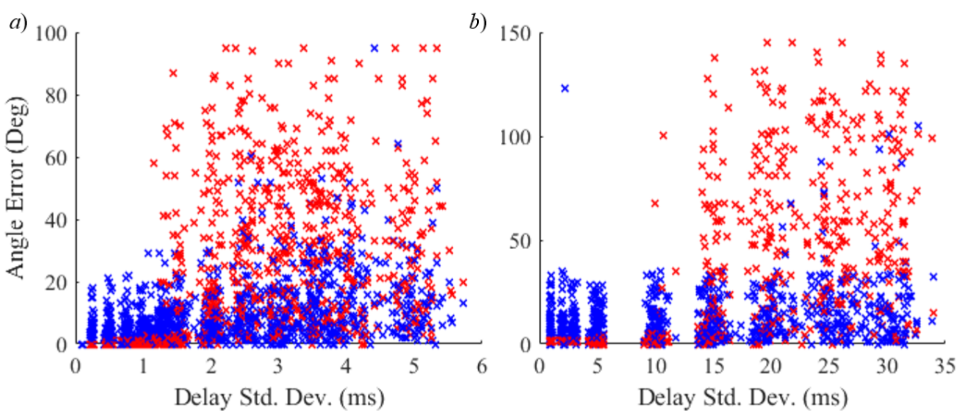

Fig. 4: Beamforming error for 100 trials of each delay std. dev. with frequency-difference (red) and conventional (blue) for a) 5m spacing, 5° arrival and b) 50m spacing, 55° arrival.

VERTICAL ARRAYS IN SHALLOW WATER

Frequency-difference beamforming has proved robust in simulations, demonstrated further in an analysis not included in this proceeding [12]. Here, experimental data from KAM11 is considered, using 100 ms, 11.2 – 32.8 kHz frequency sweeps sent in a geometry equivalent to that in Figure 2 (using the full vertical array). For the experimental case, low-frequency signals were not available for comparison with frequency-difference beamforming. Fortunately, with the source signal known, an impulse response is attainable, which can be used to construct a wide-band impulse response. This was done by locating peaks in the impulse response, recording the amplitude and timing of each peak, and generating an impulse response with a wide-band ray path corresponding to each peak. This method provides an impulse response that can be used to simulate propagation of a signal at any desired frequency. Figure 4 provides an example of this using data from KAM11. In this example, the 11.2 – 32.8 kHz sweep is used to generate an in-band conventional beamforming output (dotted black curve), which does not provide any clear direction-of-arrival indication. The wide-band impulse response described above is used to simulate propagation of a 1700-2300 Hz frequency sweep, the output of which is used for conventional beamforming over the full bandwidth at 25 Hz increments (solid blue curve). The 11.2 – 32.8 kHz sweep is used again to generate the frequency-difference beamforming output using difference frequencies averaged from 1700-2300 Hz at 25 Hz increments.

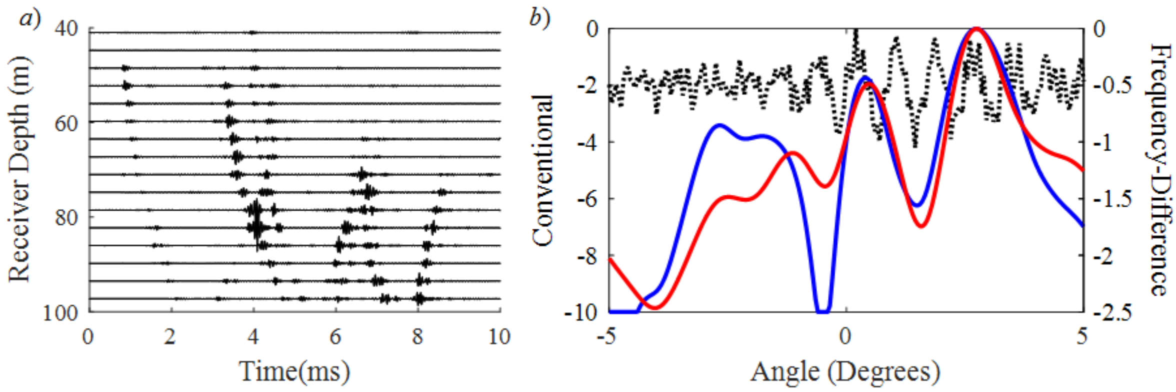

Fig. 5: a) Experimental impulse response from a source depth of 90m and b) the corresponding beamforming outputs for the simulated 1.7-2.3 kHz frequency sweep with conventional beamforming (dotted black line) and the experimental 11.2-32.8 kHz frequency sweep with conventional beamforming (blue line) and frequency-difference beamforming (red line). For the purposes of comparison, different axes are used on either side of the beamforming plot to accommodate different dynamic range.

This example clearly demonstrates that the frequency-difference beamforming with the actual measured data produces an output comparable to that which would be expected from conventional beamforming with a lower-frequency signal in the same environment, with some losses to dynamic range but reliable source localization. This results are comparable for another 15 data sets obtained over two days at various source depths.

CONCLUSIONS

Frequency-difference beamforming has been shown to work well in sparse-array conditions where conventional beamforming methods struggle. Simulations in a downward-refracting shallow water environment with a horizontal array support this conclusion and demonstrate the robustness of the frequency-difference method where conventional methods fail. Additionally, data from the KAM11 experiment supports the robustness of frequency-difference beamforming based on an expected, lower-frequency conventional beamforming output.

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