DETERMINING THE UNDERWATER ACOUSTIC PROPERTIES OF MATERIALS, WITH A FLUID-FILLED IMPEDANCE TUBE AND TWO SPATIAL-MODES REDUCED-ORDER MODEL

Yoav Vered[a], Izhak Bucher[a]

[a] Dynamics Laboratory, Faculty of Mechanical Engineering, Technion IIT, Technion City, Haifa, 3200003, Israel.

Contact: Yoav Vered, Fax no.:+972 (0)4 829 5711, E-mail: syoavv@technion.ac.il

Abstract: *In this work, a novel method to characterize the underwater acoustics properties of small specimen is being derived. The method is based on the use of a reduced-order model, which is able to capture two or more propagating modes. A separation technique is used to determine the phase velocity and the complex amplitudes of the incident and reflected waves of each mode. Currently, standard tests of acoustic impedance in air-filled tubes neglect the fluid-structure interaction (FSI) under the assumption that the enclosure solid acoustic admittance can be neglected compared to that of air at low frequency. This assumption has been investigated in fluid-filled tubes and was found to be inaccurate. It was shown that at least two propagating modes are present in the tube at all frequencies. Our proposed method uses a finite liquid-filled elastic-walled tube, which is actuated at both ends and between them a specimen is located. We aim to measure the sound field along the tube axis using an array of hydrophones. The method comprises three steps: The first step is the separation of phase velocities of each participating mode under consideration. The second step is the separation of the incident and reflected wave complex amplitudes. The third step uses a Transfer Matrix based method to determine the specimen acoustic properties using the previously estimated parameters. A numerical experiment of a coupled (FSI) tube is carried to analyse the method sensitivity to measurement noise and hydrophones bias.*

Keywords: *Impedance Tube, Fluid-Structure Interaction, Acoustic Reflection, Acoustic Transmission.*

1. INTRODUCTION

Impedance Tubes have been used since the end of the nineteenth century, and are one of the most common experimental setups to characterize specimen underwater acoustic properties. Most of the methods used to characterize acoustic properties neglect the coupled dynamics of the fluid and its enclosure, in an attempt to avoid complicated models by using rules of thumb, simplifications and various limitations [1].

Past analytical research efforts [2], [3] investigated the fluid-filled infinite-length elastic cylinder and the fluid-structure interaction (FSI). They have found that at the entire frequency range, normal modes do exist. Lafleur and Shields [4] investigated the FSI effects at the low frequency range. They concluded that at all frequencies, at least two propagation modes are present.

Various estimation methods have been developed over the years, most of them decomposed the incident and reflected pressure waves to estimate the acoustic impedance. The Multi-Point method [5], decomposes the waves with the use of the Least-Squares parameter estimation method applied to time domain signals. The Transfer Matrix method [6], builds the specimen transfer matrix and characterizes the reflection and transmission coefficients by using the decomposed waves complex amplitudes.

In the following paper a novel, three steps method is derived, with which the specimen acoustic reflection and transmission coefficients are estimated. First, each modal phase velocity is estimated using the Phase Velocity Estimation method (PVE); next, the incident and reflected waves are decomposed using the Non-Linear Least-Squares Multi-Channel method (NLLS-MC); lastly, the reflection and transmission coefficients are estimated using the Transfer Matrix method (TM). Each of the three steps derivation is described, and followed by numerical simulation.

2. METHODOLOGIES

2.1. The Three Steps Method

The experimental system model (Fig.1) consists of two water-filled cylindrical tubes, and a specimen connected between them. On the right side of the tube connected is an actuator which is capable of driving the system at a single frequency. An array of hydrophones is spread along both tubes.

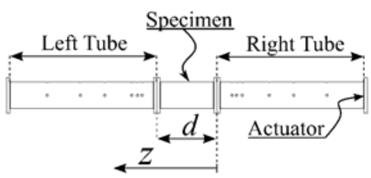

Fig. 1: Proposed experimental configuration diagram.

The reduced order model is presented herein. It takes into consideration the two spatial propagating modes present in the tubes due to FSI effects, and also considers the incident and reflected waves propagating in each tube

$$P_s(z,t) = \Re\left\{\left(\mathrm{A}_s e^{jk_1 z} + \mathrm{B}_s e^{-jk_1 z} + \mathrm{C}_s e^{jk_2 z} + \mathrm{D}_s e^{-jk_2 z}\right)e^{-j\omega_0 t}\right\}, \quad s = r, l. \tag{1}$$

Here, s is the tube side (right or left), ω_0 is the monochromatic frequency of the actuator, k_1 and k_2 are the two modal wavenumbers, A_s and C_s are the incident wave complex amplitudes of each spatial mode, and B_s and D_s are the reflected wave complex amplitudes of each spatial mode.

The suggested estimation algorithm comprises three inner steps shown in Fig. 2, which depicts the experiment flow diagram. The first step is used to decompose the two spatial modes and estimate each modal wavenumber and phase velocity. The second step is used to decompose the incident and reflected waves and estimate the four complex amplitudes, this is done for both tubes. The third step uses the phase velocities that were estimated at the first step, and the eight complex amplitudes that were estimated at the second step, to characterize the acoustic reflection and transmission coefficients of the specimen.

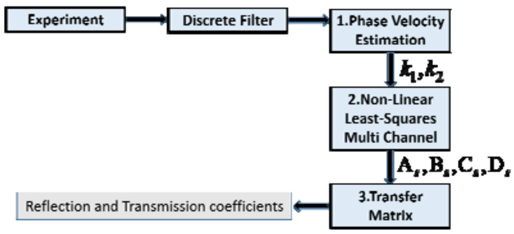

Fig. 2: The Three Step method flow-diagram.

2.2. Phase Velocity Estimation

Employing Fourier transform on Eq.(1), and defining the transfer function between the first hydrophone and all other hydrophones located at the same tube yields

$$G_{m1}(\omega) = \frac{P_r(z_1,\omega)}{P_r(z_M,\omega)}, m = 2,3,...,\mathrm{M}. \tag{2}$$

Here M is the number of hydrophones along each tube. Eq.(2) can be written as the following linear system

$$\mathbf{M}_s(k_1,k_2)\mathbf{b} = 0, \qquad \mathbf{b} = \left[\mathrm{A}_s \quad \mathrm{B}_s \quad \mathrm{C}_s \quad \mathrm{D}_s\right]^{\mathrm{T}}. \tag{3}$$

Here $\mathbf{M}_s$ is the model matrix and it depends on the two modal wavelengths, and $\mathbf{b}$ is the complex amplitudes vector. Since the system is homogenous, a non-trivial solution exists only if the model matrix is singular. Therefore, the minimization problem cost function is

$$\underbrace{\min}_{k_1,k_2} \mathrm{cond}\left(\mathbf{M}_s\right)^{-1}. \tag{4}$$

By solving the minimization problem, the two wavenumbers are estimated, and accordingly the appropriate modal axial phase velocities are calculated.

2.3. Non-Linear Least-Squares Multi Channel

Denoting the hydrophones location along each tube and the sampling time as z_m, t_n respectively, the following estimation problem can be derived from Eq.(1)

$$\begin{gathered}\mathbf{Y}_s = \mathbf{A}_s\left(k_1, k_2, g, \varphi\right)\mathbf{\Theta}_s \\ \mathbf{\Theta}_s = \begin{bmatrix} \mathrm{a}_{s,\cos} & \mathrm{a}_{s,\sin} & \mathrm{b}_{s,\cos} & \mathrm{b}_{s,\sin} & \mathrm{c}_{s,\cos} & \mathrm{c}_{s,\sin} & \mathrm{d}_{s,\cos} & \mathrm{d}_{s,\sin} \end{bmatrix}^{\mathrm{T}}_{8\times 1}\end{gathered} . \tag{5}$$

Here $\mathbf{Y}_s$ is the measurement vector, $\mathbf{A}_s$ is the model matrix which depends on the gain and phase lag of each hydrophone denoted as g and φ respectively, and the lower-case letters a, b, c, and d are the real coefficient representation of the matching capital-letters (e.g., $\mathrm{A}_s = \mathrm{a}_{s,\cos} + j\mathrm{a}_{s,\sin}$). The cost function for the minimization problem is

$$\underbrace{\min}_{\mathbf{\Theta}_s, g, \varphi} \left|\mathbf{Y}_s - \mathbf{A}_s\mathbf{\Theta}_s\right|_2^2 . \tag{6}$$

A solution for (6) is derived with the use of the Non-Linear Least-Squares method, and then the four complex amplitudes of each tube are calculated.

2.4. Transfer Matrix Method

Substituting the wavenumbers k_1 and k_2 estimated by minimizing Eq.(4), and the complex amplitudes estimated by minimizing Eq.(6) to Eq.(1), both of the specimen surfaces pressure and axial velocity can be calculated

$$\begin{gathered}\mathrm{P}\big|_{z=0} = \mathrm{A}_r + \mathrm{B}_r + \mathrm{C}_r + \mathrm{D}_r , \quad \mathrm{P}\big|_{z=d} = \mathrm{A}_l \mathrm{e}^{jk_1 d} + \mathrm{B}_l \mathrm{e}^{-jk_1 d} + \mathrm{C}_l \mathrm{e}^{jk_2 d} + \mathrm{D}_l \mathrm{e}^{-jk_2 d} \\ \mathrm{V}\big|_{z=0} = \frac{k_1}{\rho_0\omega_0}\left(\mathrm{A}_r - \mathrm{B}_r\right) + \frac{k_2}{\rho_0\omega_0}\left(\mathrm{C}_r - \mathrm{D}_r\right) \\ \mathrm{V}\big|_{z=d} = \frac{k_1}{\rho_0\omega_0}\left(\mathrm{A}_l \mathrm{e}^{jk_1 d} - \mathrm{B}_l \mathrm{e}^{-jk_1 d}\right) + \frac{k_2}{\rho_0\omega_0}\left(\mathrm{C}_l \mathrm{e}^{jk_2 d} - \mathrm{D}_l \mathrm{e}^{-jk_2 d}\right)\end{gathered} . \tag{7}$$

By following the ideas presented by Song and Bolton [6], a closed-form solution is derived

$$\begin{gathered}\mathrm{R}_a = \frac{2\left(\mathrm{P}_0\mathrm{V}_0 + \mathrm{P}_d\mathrm{V}_d\right) + \frac{k_1}{\rho_0\omega_0}\left(\mathrm{P}_0^2 - \mathrm{P}_d^2\right) + \frac{\rho_0\omega_0}{k_1}\left(\mathrm{V}_0^2 - \mathrm{V}_d^2\right) - \frac{\rho_0\omega_0}{k_1}\left(\mathrm{V}_0^2 - \mathrm{V}_d^2\right)}{2\left(\mathrm{P}_0\mathrm{V}_0 + \mathrm{P}_d\mathrm{V}_d\right) + \frac{k_1}{\rho_0\omega_0}\left(\mathrm{P}_0^2 - \mathrm{P}_d^2\right) + \frac{\rho_0\omega_0}{k_1}\left(\mathrm{V}_0^2 - \mathrm{V}_d^2\right)} \\ \mathrm{T}_a = \frac{2\left(\mathrm{P}_0\mathrm{V}_d + \mathrm{P}_d\mathrm{V}_0\right)\mathrm{e}^{-jk_1 d}}{2\left(\mathrm{P}_0\mathrm{V}_0 + \mathrm{P}_d\mathrm{V}_d\right) + \frac{k_1}{\rho_0\omega_0}\left(\mathrm{P}_0^2 - \mathrm{P}_d^2\right) + \frac{\rho_0\omega_0}{k_1}\left(\mathrm{V}_0^2 - \mathrm{V}_d^2\right)}\end{gathered} . \tag{8}$$

Here R_a and T_a are the acoustic reflection and transmission coefficients respectively and ρ_0 is the density of the fluid. Note that the proposed method differs from that of Song and Bolton since it includes the influence of the second spatial mode as indicated by the wavenumber k_2 and the four complex amplitudes C_r, D_r, C_l and D_l in Eq.(7).

3. RESULTS

A numerical simulation that uses the reduced order model (Eq.(1)), to simulate the pressure field in one of the tubes was devised. Predefined coefficients and wavenumbers were chosen for the simulation. Gaussian measurement noise was added to the simulation, additionally each hydrophone gain and phase lag were randomly chosen. The simulated signals were then passed through a digital filter. Since the signals were monochromatic the Time Domain Averaging filter [7] was used. Then, the PVE method was employed on the filtered signals, from which the two axial phase velocities were estimated. Afterwards, the NLLS-MC method was employed to estimate the four complex coefficients.

3.1. Phase Velocity Estimation

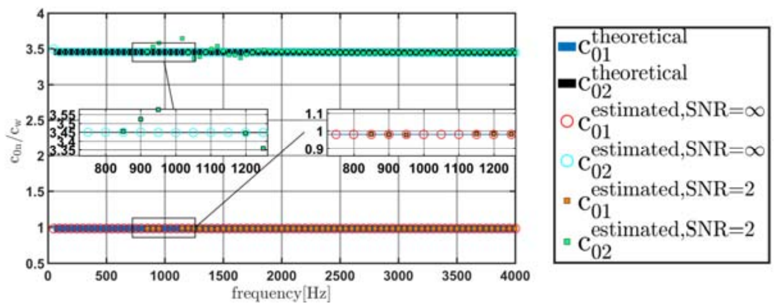

Fig. 3: Theoretical and estimated dispersion curves. Normalized by water speed of sound.

Each modal axial phase velocity ($c_{0n} = \omega_0 / k_n$, $n = 1,2$) is estimated from the numerical simulation by employing the PVE method. The dispersion curves are computed from both the chosen parameters and the estimated parameters. From the simulation results (Fig.3) it is noticeable that in the absence of noise, the PVE method is unbiased. Moreover, the estimation is accurate at the zero-frequency limit. However, in the presence of noise the estimation method shows deteriorated accuracy at low frequencies, and at some of these frequencies the estimation is unsuccessful. This happened due to a large variance in each wavelength estimation, which does not meet our imposed convergence criteria. Observing the dispersion curves of a typical fluid-filled tube [4] it can be argued that the wavenumber changes slowly at the low frequency range. Therefore, to overcome the estimation problem at low frequencies, the dispersion curves can be estimated at higher frequencies, where the convergence criteria is met, and then extrapolated to the lower frequency range.

3.2. Non-Linear Least-Squares Multi Channel

The numerical simulation and the two modal phase velocities that were estimated by the PVE method are used to estimate the model four complex amplitudes. The simulation results (Fig.4) shows that the NLLS-MC method is unbiased when no noise is present. When noise is present, the estimation is no longer exact. Moreover, the estimation does not converge at lower frequencies which is due to the PVE method convergence problem. Additionally, the estimation slightly deviates. However, the estimation falls into an acceptable tolerance by any engineering standard of less than 0.15 decibels.

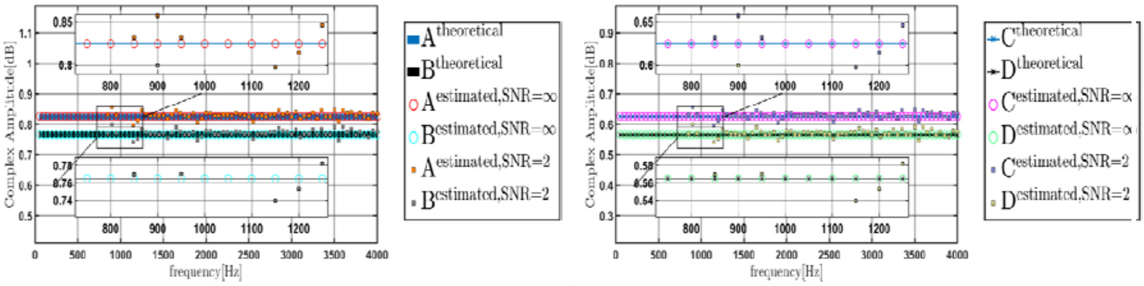

Fig. 4 : Theoretical and estimated complex amplitudes.

4. CONCLUSIONS

The proposed PVE is demonstrated to be unbiased, and can be used to decompose the spatial interference pattern, and to find each modal axial phase velocity. The proposed NLLS-MC is shown numerically to be unbiased as well, and it can be used to decompose the incident and reflected waves of each mode. In the presence of measurement noise, an alternative methodology is needed to be employed for the lower frequency range. Therefore, the dispersion curves will be estimated at higher frequencies, and then extrapolate to lower frequencies. The proposed TM method is capable of explicitly characterizing the specimen acoustic coefficients. This is done after the reduced order model is fitted to describe the wave propagations in the physical system.

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