COHERENT MATCHED-FILTER SURFACE REFLECTION LOSS AS A FUNCTION OF PULSE DURATION AND ENSONIFIED EXTENT

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Abstract: *In many applications of active sonar the propagation paths connecting a source, object of interest, and receiver include reflection from the ocean surface. A classic result describes the coherent surface reflection loss (SRL) of an acoustic pulse solely as a function of the Rayleigh roughness parameter by requiring in effect a large ensonified area (LEA) and a pulse short in duration relative to the surface wave period. At the other extreme are results accounting for motion of the surface throughout pulse reflection, but require in effect a small ensonified area (SEA) where less than a correlation length of the surface contributes to the reflection. The pulse compression achieved by matched filtering broadband pulses provides the SEA while using a pulse that may be long in duration relative to the surface-wave period. The coherent SRL for a medium-ensonified-area (MEA) is derived in this paper by extending the SEA results to account for the combination of multiple correlated components of the surface contributing to specular reflection. The derivation adds a dependence on both the ensonified area and the spatial correlation function. The model is seen to simplify to the classic LEA result when the ensonified area is large relative to the correlation length. An example evaluation of SRL using a Pierson-Moskowitz wave spectrum is presented as a function of pulse duration and bandwidth to illustrate how the loss is near the LEA result for low bandwidths and tends to the SEA result as bandwidth increases.*

Keywords: *surface reflection loss, broadband waveforms, pulse duration*

1. INTRODUCTION

Reflection of an active-sonar pulse off a rough surface results in a loss that adversely impacts detection performance. The classic models for predicting the coherent surface reflection loss (SRL) assume the pulse is short in duration relative to surface motion, so the surface can be assumed static or frozen, and that the ensonified area is large in effect relative to the correlation length of the surface [1], [2, Sect. 13.2.1]. SRL for the classic model, which will be referred to as having a large-ensonified-area (LEA), depends solely on the Rayleigh roughness parameter (γ_r), which consolidates the effects of the surface roughness, grazing angle, and acoustic wavelength. Extensions to account for pulses with a duration such that the surface is in motion throughout the reflection exist (e.g., [3–5]), but retain the LEA assumption and focus on continuous wave (CW) pulses. Recent experimental data [6] and theoretical modeling [7, 8] examined the case of broadband pulses with a duration on par with or exceeding the surface wave period so the surface is in motion throughout the reflection. Matched filtering the broadband pulses, which produce very narrow sonar resolution cells in the down-range dimension, led to assuming that only a small part of the surface contributed to the coherent reflection (i.e., in effect a small ensonified area or SEA) in [7, 8], where SRL was derived as a function of the Rayleigh roughness parameter and the product of pulse duration and the surface-wave frequency. The topic of this paper is the region between the SEA and LEA conditions where a medium-ensonified-area (MEA) implies more than one component of the surface will contribute to the coherent reflection, but not so many as to satisfy the LEA assumption.

2. REFLECTION LOSS FOR SMALL AND LARGE ENSONIFIED AREA

If an acoustic pulse $p_0(t)$ is incident on a surface with time-varying height $Z(t)$ at grazing angle θ_g (from horizontal), the reflected pulse in the specular direction can be described by applying a time-varying delay to the incident pulse,

$$p_r(t) = a_r p_0\big(t - \tau_0 + 2c_w^{-1} Z(t) \sin\theta_g\big)\,, \tag{1}$$

where a_r accounts for spreading loss, τ_0 is a bulk propagation delay from the source to the receiver along the surface-reflected path and c_w is the speed of sound. The functions $p_0(t)$ and $p_r(t)$ are assumed to represent the analytic signal of the incident and reflected pressures. Applying a matched filter (MF) to the reflected pulse results in

$$x(\tau) = \int_{-\infty}^{\infty} p_0(t) p_r^*(t+\tau)\,\mathrm{d}t. \tag{2}$$

Assuming the peak of the MF occurs at the same time delay as for a continually flat surface (i.e., $Z(t) = 0$ so the MF peak occurs when $\tau = \tau_0$), the loss in the MF relative to reflection from a continually flat surface is then the average squared modulus of $x(\tau_0)$ normalized by the peak for a flat surface,

$$\bar{L} = \frac{E[|x(\tau_0)|^2]}{D^2 a_r^2 a_0^4}, \tag{3}$$

where the incident pressure $p_0(t)$ is assumed to have a constant envelope a_0 over a duration D. This MF loss factor, which describes the effect of mismatch between the transmitted and reflected pulses, can be considered 'coherence' for sinusoidal pulses or the squared reflection coefficient of the rough surface. When converted to decibels via $-10\log_{10}\bar{L}$ it represents an increase to the transmission-loss term in the sonar equation for a single surface reflection.

The classic LEA result [1], [2, Sect. 13.2.1] assumes the pulse is short in duration relative to the surface motion so $Z(t) = Z$ is constant throughout the reflection. When the ensonified area is large enough that many different heights contribute to the reflection, the reflected pulse can be modeled by taking the expected value over the random surface displacement Z,

$$p_r(t) = E_Z\left[a_r p_0\left(t - \tau_0 + 2c_w^{-1} Z \sin\theta_g\right)\right]. \quad (4)$$

When the surface displacement is Gaussian distributed with variance σ_z^2, the MF loss is then $\bar{L}(\gamma_r) = e^{-\gamma_r^2}$ where $\gamma_r = 2k_c\sigma_z \sin\theta_g$ is the Rayleigh roughness parameter and $k_c = 2\pi f_c/c_w$ is the wavenumber at the center frequency (f_c) of the pulse.

The SEA result was derived in [7,8] under the assumption that only one small portion of the surface contributed to the reflection in order to retain the dependence on pulse duration with respect to the surface motion. The approach assumed the SEA arose from matched-filtering a large bandwidth, linear-frequency-modulated (LFM) pulse to produce a narrow down-range extent in the sonar resolution cell (approximately $c_w/(W\cos\theta_g)$ for small grazing angles where W is the bandwidth). Other factors (e.g., beamforming or Fresnel-zone width) can limit the ensonified area in the cross-range dimension. Because of the focus on SRL after matched filtering, the ensonified area is considered to be only that contributing to the reflection. This can be significantly smaller than the physically ensonified area for broadband pulses. The analysis in [7, 8] modeled the surface-wave field as a temporally bandpass space-time random process with dominant frequency f_w,

$$Z(t,\vec{x}) = \mathcal{A}(t,\vec{x})\cos(2\pi f_w t + \phi(t,\vec{x})) \quad (5)$$

where $\vec{x} = (x, y)$ represents the down-range (x) and cross-range (y) dimensions. The amplitude $\mathcal{A}(t,\vec{x})$ and phase $\phi(t,\vec{x})$ were assumed to vary slowly in time relative to the pulse duration. Under a narrowband assumption of a small bandwidth-to-center-frequency ratio (i.e., $W \ll f_c$) for the acoustic pulse, the SEA MF loss for coherent reflection was found to be

$$\bar{L}_1(\gamma_r, Df_w) = \frac{e^{-\gamma_r^2}}{D}\int_{-D}^{D}\left(1 - \frac{|\tau|}{D}\right) e^{\gamma_r^2\cos(2\pi f_w\tau)}\,\mathrm{d}\tau \quad (6)$$

which depends only on the Rayleigh roughness and the pulse duration in terms of the number of wave periods, Df_w. Asymptotically as $Df_w \to \infty$ the loss tends to

$$\bar{L}_1(\gamma_r, \infty) = e^{-\gamma_r^2} I_0\left(\gamma_r^2\right), \quad (7)$$

where $I_0(\cdot)$ is a modified Bessel function with order zero. The result in (7) is also that attained when Df_w is equal to a natural number.

3. MEDIUM ENSONIFIED AREA

The SEA requirement of the model developed in [7, 8] assumes the reflected pulse is only affected by a single surface facet, which might only occur for very large bandwidth waveforms and for particular scenarios where the effective ensonified cross-range extent is less than the correlation length of the surface in that direction. At the other extreme, the classic result assumes a static surface and a large enough ensonified area that the reflected pulse can be described by an average or expectation over all possible heights. Many scenarios of interest will be between these extremes in terms of the number of different surface heights contributing to the reflected pulse and the pulse duration relative to the surface wave period. Suppose that the ensonified area of the surface can be described as containing n time-varying heights, $Z_i(t)$ for $i = 1, \ldots, n$, contributing to specular reflection. The pulse coherently reflected from the surface is then

$$p_r(t) = \frac{a_r}{n} \sum_{i=1}^{n} p_0\big(t - \tau_0 + 2c_w^{-1} Z_i(t) \sin\theta_g\big) \tag{8}$$

where the $1/n$ scaling implements the expectation found in (4) and also ensures (8) tends to the flat-surface result (i.e., $a_r p_0(t - \tau_0)$) as the Rayleigh roughness tends to zero.

Applying a matched filter to (8) and evaluating it at the delay time (τ_0) for the peak intensity in the flat surface results in the average $x(\tau_0) = n^{-1} \sum_{i=1}^{n} x_i(\tau_0)$ where $x_i(\tau_0)$ is the matched-filter response to the ith height contributor at τ_0. The model used in [7, 8] assumed the surface at a single point could be represented as a bandpass Gaussian random process that was narrowband over the temporal extent of the pulse as described in (5). In order to account for the interaction between two different points on the surface when $|x(\tau_0)|^2$ is formed, the narrowband nature of the process needs to be extended to the spatial dimension as well. This results in the surface model

$$Z(t, \vec{x}) = \mathcal{A}(t, \vec{x}) \cos(\omega_w t + \vec{k}_w \cdot \vec{x} + \phi(t, \vec{x})), \tag{9}$$

where $\omega_w = 2\pi f_w$ and $\vec{k}_w$ is a wavenumber vector that is related to f_w through the appropriate wave dispersion equation. The time-varying surface displacement at the location of the ith height contributor ($\vec{x}_i$) is then

$$Z_i(t) = Z(t, \vec{x}_i) \approx \sqrt{2}\sigma_z A_i \cos(\omega_w t + \vec{k}_w \cdot \vec{x}_i + \phi_i) \tag{10}$$

where it is assumed that $\mathcal{A}(t, \vec{x}_i) \approx \sqrt{2}\sigma_z A_i$ and $\phi(t, \vec{x}_i) \approx \phi_i$ throughout the duration of the pulse. Following [7, 8] with a narrowband assumption with respect to the acoustic signal while using (10) in (8) to form the MF response at τ_0 to the ith height contributor results in

$$x_i(\tau_0) \approx a_r^* a_0^2 \int_0^D e^{-j\sqrt{2}\gamma_r A_i \cos\left(\omega_w t + \vec{k}_w \cdot \vec{x}_i + \phi_i\right)} \, \mathrm{d}t. \tag{11}$$

If it is then assumed that the surface height displacement is Gaussian distributed with zero mean and variance σ_z^2, the amplitude A_i will be unit-power Rayleigh distributed and the phase ϕ_i will be uniformly distributed on $(0, 2\pi)$ and independent of A_i.

In general, the amplitude-phase pair (A_i, ϕ_i) for the ith contributor is not independent of the jth, (A_j, ϕ_j), for $i \neq j$. The MF loss, which is the expected value of $|x(\tau_0)|^2$ normalized by the flat-surface result, under this general assumption can only be simplified to

$$\bar{L}(\gamma_r, Df_w) = \frac{E[|x(\tau_0)|^2]}{D^2 a_r^2 a_0^4} = \frac{1}{n^2} \sum_{i=1}^{n} \sum_{j=1}^{n} C(\rho_{ij}, Df_w, \gamma_r), \tag{12}$$

where $C(\rho, Df_w, \gamma_r)$ captures the interaction between of each pair of height contributors. Assuming the surface height displacement is a wide-sense-stationary (WSS), zero-mean, Gaussian-distributed space-time random process, it can be shown that

$$C(\rho, Df_w, \gamma_r) = \frac{e^{-\gamma_r^2}}{D} \int_{-D}^{D} \left(1 - \frac{|\tau|}{D}\right) e^{|\rho|\gamma_r^2 \cos(2\pi f_w \tau + \angle\rho)} \, \mathrm{d}\tau, \tag{13}$$

where ρ is the value of the analytic signal form of the normalized spatial correlation function along the line between $\vec{x}_i$ and $\vec{x}_j$ evaluated at the separation distance between the two points. Note that this only differs from the SEA result in (6) by the ρ within the exponent inside the integral and the $\angle\rho$ adjusting the phase of the cosine. When $\rho = 1$, (13) simplifies to the SEA result, $C(1, Df_w, \gamma_r) = \bar{L}_1(\gamma_r, Df_w)$, because the two surface heights are identical and fully correlated and therefore satisfy the SEA assumptions. Conversely, when $\rho = 0$, (13) produces the LEA result, $C(0, Df_w, \gamma_r) = e^{-\gamma_r^2}$ irrespective of Df_w.

3.1. One-dimensional spatial correlation function

Consider a 1-D scenario with a length X_e of the surface ensonified by the pulse and contributing to the coherent reflection. If each point of the surface that contributes to the reflection is assumed to occur randomly within the effectively ensonified extent, it can be shown that

$$\bar{L}(\gamma_r, Df_w, X_e) = \frac{e^{-\gamma_r^2}}{X_e D} \int_{-X_e}^{X_e} \int_{-D}^{D} \left(1 - \frac{|x|}{X_e}\right)\left(1 - \frac{|\tau|}{D}\right) e^{\gamma_r^2|\rho(x)|\cos\left[2\pi f_w \tau + \angle\rho(x)\right]} \,\mathrm{d}\tau\,\mathrm{d}x \quad (14)$$

Thus, the MF loss in the MEA scenario depends on the Rayleigh roughness, pulse duration in terms of surface wave periods, and the SCF in conjunction with the ensonified extent. To indicate the dependence on the SCF and the ensonified extent, X_e is added to $\bar{L}$ as an independent variable with the dependence on the SCF an implicit assumption.

While (14) requires evaluation of a double integral, it simplifies to a single integral when $Df_w = k$ is an integer or for the asymptotic case of large-Df_w,

$$\bar{L}(\gamma_r, k, X_e) = \frac{e^{-\gamma_r^2}}{X_e} \int_{-X_e}^{X_e} \left(1 - \frac{|x|}{X_e}\right) I_0\left(\gamma_r^2|\rho(x)|\right)\,\mathrm{d}x. \quad (15)$$

Based on the results of [7, 8], (15) should be a useful approximation when $Df_w \geq 1$. As expected, both (14) and (15) simplify to their respective SEA result when $X_e \to 0$ and to the LEA result when $X_e \to \infty$ (the latter assumes $|\rho(x)|$ eventually decays to zero as $|x|$ increases).

4. EXAMPLE

Consider a Pierson-Moskowitz wave spectrum [9, Sect. 16.4] restricted to the 1-D scenario. For a specific example, consider using a broadband sonar pulse with a center frequency of 3 kHz and a sensing geometry such that the grazing angle is 4°. Using the RMS roughness of the Pierson-Moskowitz wave spectrum with a 10-m/s wind speed, which is approximately sea-state 4, this scenario results in a Rayleigh roughness of $\gamma_r = 0.98$. From the perspective of pulse design and sonar performance analysis, one desires to obtain SRL as a function of pulse duration and bandwidth given the aforementioned sensing scenario. This is shown in Fig. 1 as a function of pulse duration for a variety of bandwidths ranging from 5 Hz to 500 Hz.

The limiting conditions of the SEA and LEA results are shown as black dashed lines. The LEA result shows no dependence on pulse duration; however, the CW-pulse result (shown as a black dash-dotted line and obtained by assuming $W = 1/D$) represents a more realistic upper bound on SRL by acknowledging that short-duration pulses may not ensonify a large enough area to satisfy the LEA model assumptions. SRL for the 5-Hz-bandwidth pulse is very close to that of the CW pulse and is identical when the duration is less than 0.2 s because the pulse duration then dictates the ensonified extent as opposed to the bandwidth. As the pulse bandwidth increases, for a given pulse duration, the effective ensonified extent decreases and the SRL tends toward the SEA result with the 500-Hz-bandwidth pulse nearly achieving it.

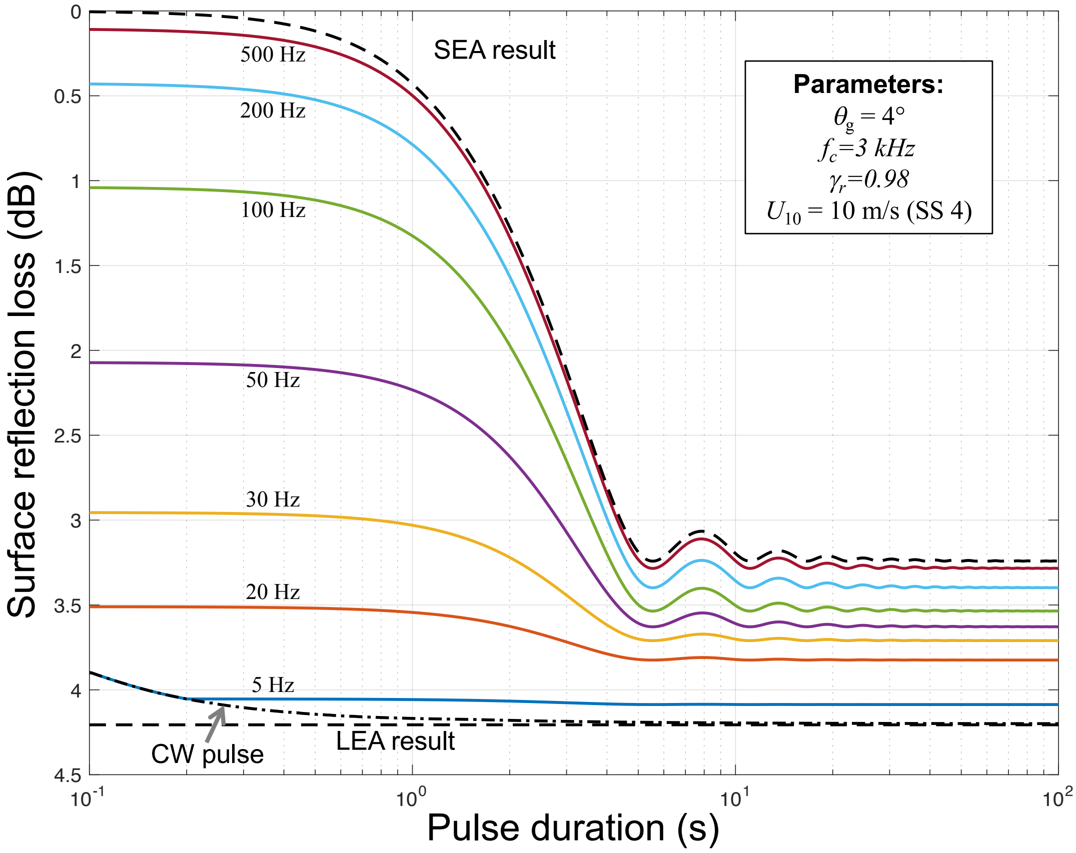

Fig. 1: Example calculation of SRL for a 1-D Pierson-Moskowitz wave-height spectrum.

5. CONCLUSIONS

The focus of this paper has been on evaluating the loss incurred after matched filtering a pulse coherently reflected from a Gaussian-distributed rough ocean surface that is in motion throughout the temporal extent of the pulse. The pulse-compression effect of matched filtering a broadband pulse makes the effective ensonified area inversely proportional to pulse bandwidth rather than proportional to pulse duration, consequently reducing SRL. The research presented extends earlier results restricting consideration to effective ensonification of an area smaller than the correlation length of the surface to allow an area covering multiple correlation lengths. These medium-ensonified-area results tie together the small-ensonified-area result with the classic large-ensonified-area result and allow evaluation of SRL as a function of the acoustic (Rayleigh) roughness, pulse duration relative to surface wave period, and the effective ensonified area.

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