QUANTIFYING THE IMPACT OF UNCERTAINTY ON SONAR PERFORMANCE PREDICTIONS.

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Abstract: *Sonar performance predictions are vital to the design of new sonar systems, the optimal deployment of existing systems and to the analysis of experimental data. The ocean environment has a strong influence on underwater acoustic propagation, scattering and noise-generating processes. This means that accurate prediction of sonar performance is usually achieved with computer codes that require high-quality input data describing meteorological, oceanographic and geological conditions in the environment of interest. Data describing environmental conditions are available from databases and forecasts but significant uncertainties are associated with them. These uncertainties may come from a fundamental lack of knowledge of, for example, the properties of seabed sediments. They may also stem from modelling uncertainties in forecasts that predict ocean and weather conditions. Further input-data uncertainties arise from the precision to which the problem under study is defined in terms of temporal and spatial spreads for which 'representative' sonar-performance predictions are required.*

A second source of uncertainty arises from the adversarial nature of anti-submarine-warfare scenarios for which sonar-performance calculations are required. Submerged targets will modify their behaviour to minimize detectability and any prediction of sonar performance is subject to potentially large changes arising from choices made outside the control of the sonar operator.

A methodology is described that captures uncertainties associated with input-data and tactical considerations and produces estimates of their impact on measures of sonar performance. Input-data-uncertainty impacts are quantified via a "scaled sensitivity" parameter that measures the importance of uncertainty, including all inter-parameter dependencies. Tactical uncertainties are handled using game-theoretic approaches.

Keywords: *Sonar, Uncertainty, Environmental, Game Theory.*

1. INTRODUCTION

Any lack of knowledge (uncertainty) associated with input data for sonar performance calculations causes a spread in possible values for Measures of Performance (MOPs) used to quantify sonars' utility. Studies directed at relating this 'output uncertainty' to input-data uncertainty are conceptually simple in that repeated calculations may be performed with environmental data that capture the input uncertainty. These results can be treated statistically and performance quantified using measures of 'best estimate' (e.g. mean or median) and 'spread' (e.g. standard deviation or inter-quartile range). However, concisely summarising the results of such studies – which may involve thousands of individual calculations – remains an issue.

The situation is further complicated if the sonar application under study is Anti-Submarine Warfare (ASW) which is adversarial in nature. In such situations, sonar MOPs are strongly affected by tactical choices made by the submarine target. These choices represent a 'tactical uncertainty' and, while of a fundamentally different nature from input-data uncertainty, they also add to the spread of possible values that might best describe sonar performance in a given situation. This form of uncertainty is poorly handled by the Monte-Carlo techniques used with environmental data because target choices are not well-described by random models.

In this paper we describe an approach by which both tactical and environmental uncertainty are included in sonar performance modelling studies aimed at answering two basic questions:

1. Which environmental parameter's uncertainty is the most important?
2. What are the consequences of this uncertainty?

Answers to these questions may be provided only for specific scenarios defined in terms of a place and a time of year. General conclusions that are 'portable' across different parts of the world and different seasons are not possible because the size of parameter uncertainty varies significantly with time and location. For example, the spread of possible wind-speeds within a single season will be very different in Winter and Summer in a given location and will vary with location for a given season. Furthermore, the consequences of that uncertainty will depend on the values for other environmental parameters, such as ocean sound-speed, which will also vary with time and location. Thus, single definitive answers to the two questions above are not sought. Instead, answers are provided for a particular ocean area, within separate seasons of the year.

2. TACTICAL UNCERTAINTY

Submarine targets can adapt their behaviour to minimise their chances of detection. This may involve changing course and speed to reduce target strength (TS) and relative Doppler shift. Because layers in the ocean sound-speed profile (SSP) result in detectability changing significantly with target depth, the target may also seek an operating depth that minimises its chance of detection.

	R_{det}(km)	Z_{Target} 10	20	30	50	90	
Z_{Sonar}	25	36	37	11	11	7	p=0.39
	35	8	26	24	32	28	p=0.61
	55	2	14	13	35	31	p=0
	95	2	7	3	30	34	p=0
		p=0.32	p=0	p=0.68	p=0	p=0	**19 km**

Table 1: Illustrative values for detection range for various target and sonar depths. The p-values indicate players' optimal 'investments' for each choice. The bold value in the bottom-right shows the 'value' of the game, being a sum of the p-weighted values .

Table 1 shows illustrative values for detection range in a particular environment for several choices of sonar and target depth. High values result from sonar and target being at near-matching depths and low values from mismatches. The sonar cannot ensure a large detection range without knowing the target depth and the target cannot minimise detection range without knowing the sonar depth. The optimal (minimax) solution in this case can be found by each player determining the worst possible outcome for all of his choices and attempting to minimise this 'worst case'. This is a standard procedure in game theory [1] for two-player, zero-sum games played by equally-informed, rational opponents. The solution gives 'investments' (shown by p-values in the table) that each player should place in each choice to yield the optimum return. The nature of the investment can be related to deployment of separate assets (e.g. sonobuoys) at different depths or to proportions of time to be spent at each depth by a variable-depth sonar. Choices which are always worse than an alternative receive zero weighting. The final 'value' of the game is p-weighted sum of all possible options. It represents an estimate of the likely detection range, taking into account each player's likely (but unknown) behaviour. This is a better estimate than a mean or median value over the table that would include the effects of sonar/target choices that would never be made by rational players.

In some situations, the approach above may be unnecessary and a single target depth may be assumed. This may be because of the role assigned to a sonar (e.g. to search for a deep target because another detection method will be used for shallow targets) or because a target will, for example, be forced to come to periscope depth at some stage and this is identified as being the best opportunity for detection and therefore the situation for which sonar depth should be optimised.

3. ENVIRONMENTAL UNCERTAINTY

The results shown in Table 1 are specific to one set of environmental data and the 'value of the game' represents a good estimate of the likely detection range given those data. The detection range values are specific to meteorological data (e.g. wind speed), oceanographic data (e.g. sound-speed profile), geological data (seabed type and scattering strength) and data describing the TS. None of these data are perfectly known and their uncertainty equates to an uncertainty in performance predictions.

Capturing input-data uncertainty requires knowledge of the potential spread in data and this is not always included in databases. In some cases – such as wind-speed - uncertainty can be quantified by a standard deviation value about the climatological value. In other cases – such as SSP – the uncertainty is not only rarely given but it is also impossible to quantify by a single number. Some types of data – such as seabed type – are subject to very large uncertainties because of a basic lack of human knowledge about the ocean environment.

Another type of uncertainty arises because of 'specification spread', i.e. the range of conditions that could fall within the description of the problem under study. For example, a quantification of sonar performance may be required "In the North Sea in Winter" but many sets of environmental conditions fall within this broad description and this represents another source of uncertainty.

Once input uncertainty is captured, its consequences can be predicted by Monte-Carlo methods producing statistical descriptions from repeated calculations using input data within the uncertainty bounds. However, concise summary of these results is a challenge because the consequences of uncertainty in one parameter depend on the values chosen for other parameters. For example, the consequences of uncertainty in seabed type are different for upward-refracting SSPs than for downward-refracting. Thus, the uncertainty in each parameter has many potential impacts on the output.

In the study reported here, the relative importance of uncertainty in different parameters was quantified by the concept of "scaled sensitivity" (S_s) which was a measure of "***The average consequences of varying a parameter within its uncertainty, divided by the average consequences of keeping the parameter constant and varying all others within their uncertainties***". This calculation process is shown schematically in Fig. 1.

		Case ->								
		1	2	3	...	...	...	$N=N_{Tot}/4$	**Spread**	**Avg**
Study Param.	V1	AER_{11}	AER_{12}	...	...	...	...	AER_{1N}	$\boldsymbol{\Delta_{r1}}$	
	V2	AER_{21}	...	...	...	...	...	...	$\boldsymbol{\Delta_{r2}}$	
	V3	AER_{31}	...	...	...	...	...	...	$\boldsymbol{\Delta_{r3}}$	$\boldsymbol{\langle\Delta_r\rangle}$
	V4	AER_{41}	...	...	...	...	...	...	$\boldsymbol{\Delta_{r4}}$	
	Spread	$\boldsymbol{\Delta_{c1}}$	$\boldsymbol{\Delta_{c2}}$	**...**	**...**	**...**	**...**	$\boldsymbol{\Delta_{cN}}$		
	Avg	$\boldsymbol{\langle\Delta_c\rangle}$							$S_S = \boldsymbol{\langle\Delta_c\rangle / \langle\Delta_r\rangle}$	

Fig.1: Calculation of scaled sensitivity S_s for a study parameter with 4 values as part of a study with N_{Tot} total combinations of environmental conditions.

Fig. 1 shows a situation in which sonar performance has been quantified via "area-equivalent range" (AER) which is the radius of a circle with area equal to the sonar's coverage area. AER is calculated for N_{Tot} environments covering the combined uncertainty in all input parameters. One of these has four values, V1 to V4 and the AER values are sorted into a matrix such that the study parameter varies down columns while all other parameters remain the same. The spread of AER values down a column represents the consequences of uncertainty in the parameter for one combination of all other parameters. The average over all columns represents the average consequences of the parameter uncertainty. This is scaled by the average of the spread over matrix rows, representing the average consequences of changing all other parameters while holding the study parameter constant. If the resulting S_s value is higher than 1, this indicates that the parameter uncertainty is, on average, as important as the uncertainty in all other parameters combined. S_s values approaching 2 indicate a parameter whose uncertainty is near-dominant.

4. CAPTURING INPUT SPREAD

Input spread was captured via an assessment of the uncertainties likely to be associated with a study of sonar performance in a broadly specified area (the Southwest approaches - SWAPPS) in Spring, Summer, Autumn and Winter. For each area/season combination, environmental uncertainty was captured in a manner shown in Table 2.

	Parameter Name				
	Wind-speed (m/s)	*SSP*	*Seabed Index*	*TS (dB re m²)*	*Lambert μ (dB)*
'Low' Value	**0**	Shallowest point inside 100 km square	1 Extremely Coarse Sand	5	-27
			2 Medium Sand (b)		
	$W_{clim}/2$		3 Fine Sand(b)		
↓	$\mathbf{W_{clim}}$	Central location	4 Very Fine Sand		-22
	$½(2W_{clim}+3\sigma_{ws})$		5 Very Fine Sand(b)		
		Deepest point inside 100 km square	6 Coarse Silt		
'High' Value	$\mathbf{W_{clim}+3\sigma_{ws}}$		7 Medium Silt(b)	15	-17

Table 2: Input-data Spread.

Wind-speed was taken from KNMI databases [2] of climatological wind-speed and standard deviation. These values were used to produce five values spread between zero and the climatological value plus three standard deviations. The middle value was equal to the climatological value. SSP data were taken from the World Ocean Atlas [3] at the centre point of the defined area and deviations from this were taken as the SSP at the shallowest and deepest point within 100 km of the central point. Seabed type is typically not well known and a wide variation over seven possible types was taken from ocean-acoustic literature [4]. Seabed scattering strength is an important parameter for shallow-water active sonar and this property, quantified by the Lambert-μ parameter [5], was varied by +/- 5 dB about a central value of -22 dB. Target strength was also included in the input spread, even though it is not strictly an environmental property. This was done so that the consequences of uncertainty in other parameters could directly be scaled against that of an important sonar parameter also subject to considerable uncertainty.

5. RESULTS

Environmental input data covering the uncertainties described in Table 2 were used as input to a mathematical model [6] that predicted target echo level as a function of target range, along with sources of interference such as ambient noise and reverberation [7].

These descriptors were combined with a detection threshold (DT) representing the signal-to-noise ratio (SNR) necessary for a 50% probability of detection, consistent with an acceptable level of false alarms. The total area in which the SNR was greater than the DT was then calculated and converted to an AER. This process was done once for a target at periscope depth and once for a minimax solution to a game table similar to Table 1 with target and sonar depths chosen to represent sensible deployment depths (e.g. best-detection depth, best-evasion depth, etc.) given the local SSP.

Fig. 2 shows results for a study in the SWAPPS area in four seasons. The top panel has bars showing S_s for the five uncertain parameters with blue bars indicating results of the minimax calculation and red bars showing results for a target placed at periscope depth. In each season, the most important parameter is identified as being the one with the highest S_s. It is interesting to note that this parameter changes with season and is not always the same for the minimax and periscope-depth cases. The lower panel of Fig. 2 shows a 'reduced spread' plot where AER values are plotted as a function of the most important parameter and uncertainty due to other parameters is summarised by "bar and whisker" plots showing 25^{th}, 50^{th} (median) and 75^{th} percentiles along with outliers plotted as dots. One bar-and-whisker is plotted for each value of the most important parameter so that the results for January for the periscope-depth target have two bars (one for each TS) and five values (one for each wind-speed) for the minimax target. Y-axis markers have been removed to avoid making the data sensitive.

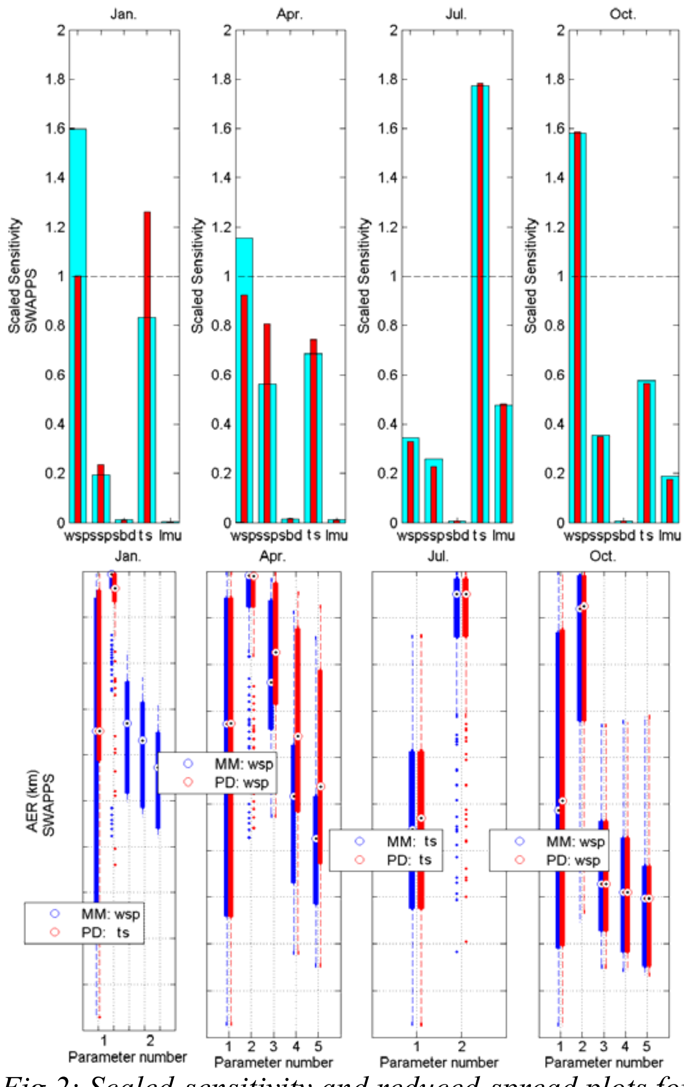

Fig.2: Scaled-sensitivity and reduced-spread plots for a-

priori planning.

The spreads in Fig. 2 are large and wind-speed is the largest source of uncertainty in more than half the cases plotted. Bar-and-whisker plots for cases where wind-speed was the most important uncertainty show an interesting feature of a maximum in AER values for small but non-zero wind-speeds. This is a consequence of the complicated dependence of detection on wind-speed which affects ambient noise, target echo level, seabed reverberation (via surface-reflection loss) and sea-surface reverberation. The AER maximum occurs for wind-speeds high enough for surface reflection loss to suppress high-angle paths responsible for seabed reverberation while low enough for sea-surface reverberation and ambient noise to remain small.

The lower panel of Fig. 2 shows dots along with the bar-and-whisker plots and these are ‘outliers’. They are defined to be values above (or below) the 75th (or 25th) percentile points by more than 1.5 times the inter-quartile range. Outliers are most commonly observed for cases where AERs ‘top out’ at the maximum value considered, corresponding to the upper limit of the y-axis. In these cases, the estimate of the interquartile range is corrupted by the topping-out and many points are identified as outliers.

In general, the spreads on AER values can remain high even when the most uncertain parameter is extracted. For example, the AER values for the lowest wind-speed in the January and April cases have an interquartile range that is more than half the size of the maximum AER considered. This represents a situation where AER is highly uncertain. In this way, the approach outlined here does not ‘solve’ the problem of uncertainty in the sense of removing uncertainty from predictions. It allows those situations for which uncertainty is high to be identified along with those where it is more manageable, such as for the highest wind-speed considered in January and April for the minimax case.

The results in Fig. 2 are representative of the situation for a-priori planning where the time before an operation is too large for a reliable weather forecast to be available. Fig. 3 shows similar plots for a situation in which the uncertainty on wind-speed has been reduced to $^+/_-$ 10% - representative of the uncertainty associated with a next-day weather forecast. Wind-speed is no longer the most important uncertainty and the spreads on the bar-and-whisker plots are now much reduced with respect to Fig. 2.

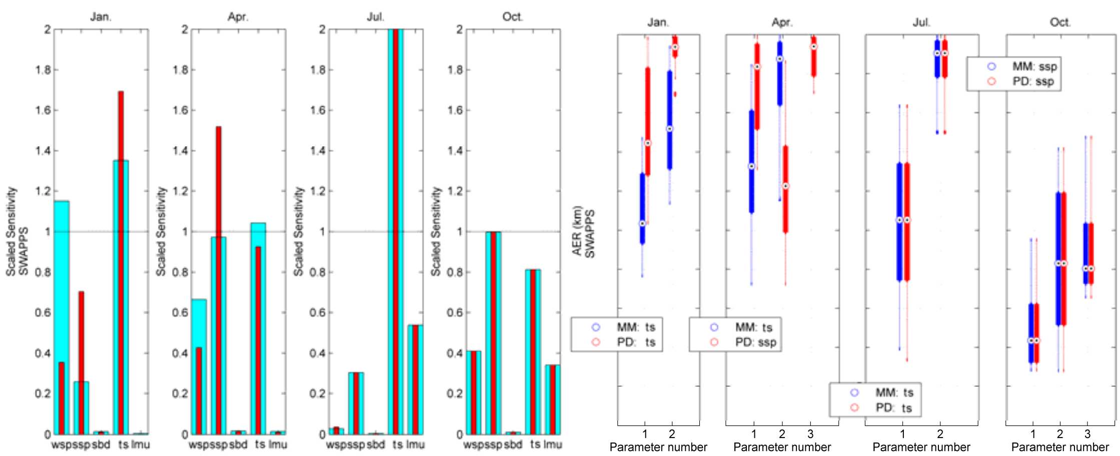

Fig.3: Scaled-sensitivity and reduced-spread plots for next-day planning.

Comparison of Fig. 2 and Fig. 3 shows how the uncertainty associated with predictions of sonar performance reduces as better environmental information becomes available. It also shows that any further reduction in wind-speed uncertainty would not strongly change the total uncertainty in AER since this is now more strongly affected by uncertainties in TS and SSP.

Knowledge of the most important sources of uncertainty allows the prioritisation of environmental-assessment activities so that effort can be directed at measuring the most important parameters. It also allows analysis of optimum tactics in terms of ‘best-case’ and ‘worst-case’ scenarios that might occur within the AER uncertainty. One final benefit of the type of uncertainty data shown in Fig. 2 and Fig. 3 is that it allows the user to ‘know what they do not know’, i.e. to be aware that the sonar-performance prediction made for best-estimate conditions is not an absolutely accurate value that can be used to plan very precise tactics or deployment options.

6. CONCLUSIONS

A method has been outlined that allows the effects of both tactical and environmental uncertainty to be summarised. The parameter whose uncertainty is most important can be identified via the concept of scaled sensitivity. The importance of this uncertainty can be shown by ‘reduced spread’ plots that show how detection range varies with the most important parameter, while uncertainty due to other parameters is summarised by bar-and-whisker plots capturing median and interquartile spreads.

For the area studied the most important parameter was shown to vary with season. Wind-speed was most frequently identified as the parameter with the most important uncertainty for cases representing predictions made months/weeks in advance when no reliable weather forecast is available. When wind-speed uncertainty was reduced to be representative of a situation where operations are to be planned for the next day, and a reliable weather forecast is consequently available, other parameters’ uncertainty was shown to be more important.

7. ACKNOWLEDGEMENTS

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