MEASUREMENT ERRORS IN ULTRASONIC DOPPLER DEVICES CAUSED BY VARIATION IN THE SPEED OF SOUND ALONG THE PROPAGATION PATH

Shokichi Tanaka, Hideyuki Nomura, Tomoo Kamakura

The University of Electro-Communications 1-5-1 Chofugaoka, Chofu-shi, Tokyo 182-8585, Japan

shokichi.tanaka@uec.ac.jp, h.nomura@uec.ac.jp, kamakura@uec.ac.jp

Abstract: *Underwater ultrasonic devices using Doppler effects assume that the speed of sound in the water is constant and model it as the average speed around the transducer. However, because the speed of sound in the sea varies with depth, this assumption has the potential to result in measurement errors. Accordingly, the present authors have previously investigated the influence of the speed of sound and the sea current on the Doppler effects and derived an improved Doppler shift equation by considering those speed and current that are dependent on the propagation path. In this study, the measurement errors caused by the assumption of a constant speed of sound used in the conventional Doppler shift equation were obtained in reference to the improved Doppler shift equation. By comparing the Doppler shifts calculated from the conventional and improved equations, the sources of measurement error arising in the conventional case were quantitatively described. The measurement error of the target speed, which is the value ultimately being measured, were evaluated using a parameter defined as the ratio of the speed of sound near the source to that near the target. It was found that the relative error of the target speed is approximately inversely proportional to this ratio. It was also demonstrated that this error is a systematic error in ultrasonic Doppler devices.*

Keywords: *Doppler effects, speed of sound, improved Doppler shift equation, Doppler devices, measurement errors.*

1. INTRODUCTION

Generally, underwater ultrasonic devices using Doppler effects, such as Doppler sonar devices and acoustic Doppler current profilers (ADCPs), model the speed of sound in water as the average speed around the transducer [1], [2]. However, because the speed of sound in the sea varies with the water depth, assuming a constant speed of sound may produce measurement errors. Therefore, an improved Doppler shift equation has been previously derived by the present authors by considering the dependence of the speed of sound and the sea current on the propagation path [3]. Based on the derived improved Doppler shift equation, the influence of the position dependence of the speed of sound and the sea current on Doppler effects has been investigated. In addition, the validity of the improved Doppler shift equation has been experimentally verified [3].

In the present study, measurement errors arising in the conventional equation were examined by comparing the conventional and improved Doppler shift equations.

2. IMPROVED DOPPLER SHIFT EQUATION

This section outlines the improved Doppler shift equation derived in a previous study [3]. As shown in Fig. 1, a one-dimensional model was considered such that sound source S and target T are located along a straight line. Fig. 1 also presents the variable definitions used here. Note that both the current speeds u_s and u_t, and the sound speeds c_s and c_t are considered to be functions of position only.

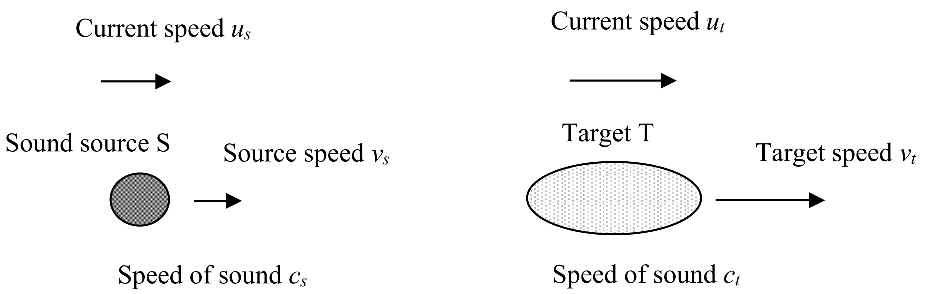

Fig. 1: One-dimensional model of sound source S and target T.

The Doppler shift Δf_{old} obtained from the conventional equation used in present ultrasonic Doppler devices is given as [1], [2]

$$\Delta f_{old} = 2\frac{(v_s - v_t)}{c} \cdot f\,, \tag{1}$$

where c is the speed of sound and f is the transmitted frequency. The speed of sound c is usually considered to be the average speed of sound around the sound source:

$$c = c_s. \tag{2}$$

It was then assumed that $(c_s, c_t) \gg (v_s, v_t, u_s, u_t)$, and a first-order approximation was derived. When the speed of sound and the sea current vary with the position along the propagation path, the improved equation for the Doppler shift Δf_{new} is given as [3]

$$\Delta f_{new} = 2\left(\frac{v_s}{c_s} - \frac{v_t}{c_t}\right)f \quad (3)$$

The following conclusions were drawn from (3).

(i) Only the speeds of sound c_s and c_t affect the Doppler shift.
(ii) The speeds of the sea currents u_s and u_t do not generally affect the Doppler shift.
(iii) The speeds v_s and v_t of the source and target are coupled to the corresponding speeds of sound c_s and c_t in the surrounding water. Therefore, when v_s is zero, the Doppler shift is independent of c_s. Similarly, when v_t is zero, the Doppler shift is independent of c_t.
(iv) When the speed of sound is constant ($c = c_s = c_t$), (3) is consistent with (1).

When the one-dimensional model is not satisfied (i.e. the direction of v_s is not coincident with that of the propagating wave), the right-hand sides of (1) and (3) must be multiplied by $\cos\theta$ to correct for the direction θ of motion of the sound source relative to that of the wave propagation.

3. EVALUATION OF ERRORS

The conventional Doppler shift equation (1) was evaluated by comparing it with the improved equation (3). When v_s, v_t, c_s, and c_t are given, Δf_{new} in (3) is considered to represent the true Doppler shift. The speed v_s of the sound source is also considered to be known. Therefore, the value to be measured is the speed of the target. The measured target speed v_{tm} was calculated using (1), and the difference between v_{tm} and the true value v_t was obtained as the measurement error.

The parameter A was defined as the ratio of c_t to c_s:

$$A \equiv \frac{c_t}{c_s} \quad (4)$$

First, the error of the Doppler shift was evaluated. The difference between the Doppler shifts obtained using (1) and (3) is given by

$$\Delta f_{dif} = \Delta f_{old} - \Delta f_{new} = 2f\frac{v_t}{c_s}\left(-1+\frac{1}{A}\right) \quad (5)$$

Then, the relative error of Δf_{old} is obtained as

$$\mathrm{RE}(\Delta f_{dif}) = \frac{\Delta f_{dif}}{\Delta f_{new}} = \frac{-v_t \cdot A + v_t}{v_s \cdot A - v_t} \quad (6)$$

Next, the error of the speed of the target was evaluated. Under the assumption that the measured Doppler shift is Δf_{new}, the target speed v_{tm} is given based on (1) as

$$\Delta f_{new} = 2\frac{(v_s - v_{tm})}{c_s} \cdot f \quad (7)$$

From (7) and (3), v_{tm} was obtained as

$$v_{tm} = \frac{v_t}{A}. \tag{8}$$

Therefore, the error of the moving speed of the target v_{tdif} was obtained as

$$v_{tdif} = v_{tm} - v_t = \left(\frac{1}{A} - 1\right) v_t. \tag{9}$$

From this, the relative error of the moving speed of the target is given as

$$\mathrm{RE}(v_{tdif}) = \frac{v_{tdif}}{v_t} = \frac{1}{A} - 1. \tag{10}$$

Eqs. (9) and (10) indicate that the error of the target speed arises when the magnitudes of c_s and c_t differ ($A \neq 1$). This error is considered to be a systematic error.

4. NUMERICAL RESULTS OF ERRORS

In this section, the numerical results of the errors obtained in several cases are shown. It was assumed that $A \neq 1$ as a premise. First, the relative error of the Doppler shift given by (6) was evaluated. The following conclusions were drawn from (6).

(i) When $v_s = 0$ and $v_t \neq 0$, $\mathrm{RE}(\Delta f_{dif}) = A - 1$. In this case, the relative error of the Doppler shift changes in proportion to the value of the ratio A.
(ii) When $v_s \neq 0$ and $v_t = 0$, $\mathrm{RE}(\Delta f_{dif}) = 0$. In this case, no error in the Doppler shift occurs, regardless of the value of the ratio A.
(iii) When $v_s = v_t \neq 0$, $\mathrm{RE}(\Delta f_{dif}) = -1$. In this case, the error of the Doppler shift is constant (-1) regardless of the value of the ratio A.
(iv) In the case where the denominator of (6) is zero, $\mathrm{RE}(\Delta f_{dif})$ diverges to $\pm\infty$.

Next, the relative error of the moving speed of the target given by (10) was evaluated. Fig. 2 shows $\mathrm{RE}(v_{tdif})$ calculated from (10) plotted against A. It was found from Fig. 2 that $\mathrm{RE}(v_{tdif})$ decreases approximately in inverse proportion to A. That is, when the speed of sound c_t around the target is different from that c_s around the sound source, the relative error $\mathrm{RE}(v_{tdif})$ of the target speed is of the same order of magnitude as the ratio A.

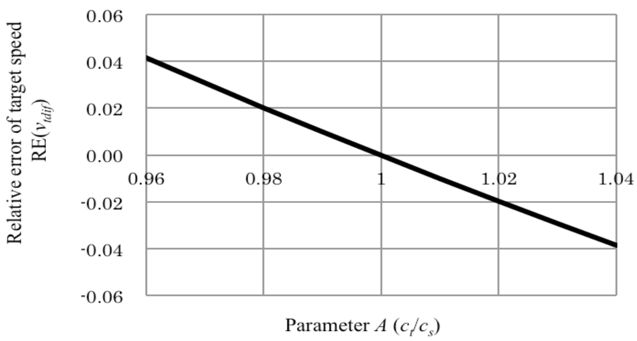

Fig. 2: Relative measurement error of target speed $\mathrm{RE}(v_{tdif})$ *plotted against the ratio A.*

5. MEASUREMENT ERRORS OF DOPPLER DEVICES

Normally, as the depth from the sea surface increases, the speed of sound decreases, mainly because of the decrease in the water temperature. Although this decrease varies from position to position in the sea, the difference between the speed of sound near the sea surface and that at some depth below the surface may differ by only 1% to 2%. Thus, the measurement error of the flow velocity at the relevant water depth can be expected to be approximately 1% to 2%.

Furthermore, when transmitting a sound beam at a depression angle θ, as with an ADCP, and measuring the horizontal component of the current velocity, the influence of the bending of sound waves is also superimposed. The measurement error in this case can be obtained as described below.

Generally, the distribution of the speed of sound in the sea is considered to have a stratified structure in the depth or vertical direction. Fig. 3 shows the geometry of the bending of sound waves when the structure of the sea is multiple layers and the variable definitions, where c_s (the speed of sound in the surface layer in contact with the sound source) > c_2 > ... > c_t (the speed of sound in the layer containing the target). The angle of incidence of the sound with respect to the surface layer is denoted θ_s and the angle at which it strikes the target is θ_t. The relationships between the angles of refraction at each layer boundary are given in accordance with Snell's law as

$$\frac{\cos\theta_s}{c_s} = \frac{\cos\theta_2}{c_2} = \cdots = \frac{\cos\theta_t}{c_t} \quad . \tag{11}$$

If only the Doppler shift component in the horizontal plane with respect to the direction of the sound beam is considered, the horizontal components v_s' and v_t' of the speeds of the sound source and target are given respectively as

$$v_s' = v_s \cdot \cos\theta_s , \tag{12}$$

$$v_t' = v_t \cdot \cos\theta_t \; . \tag{13}$$

Therefore, the equation for the Doppler shift Δf, which describes the case of a sound beam transmitted at a depression angle of θ and the horizontal component of the current velocity is measured, is given as

$$\Delta f = 2\left(\frac{v_s'}{c_s} - \frac{v_t'}{c_t}\right) \cdot f = 2\left(\frac{v_s \cdot \cos\theta_s}{c_s} - \frac{v_t \cdot \cos\theta_t}{c_t}\right) \cdot f = 2\left(\frac{v_s - v_t}{c_s}\right) \cdot \cos\theta_s \cdot f \quad . \tag{14}$$

Eq. (14) has exactly the same form as the conventional Doppler shift equation using only the speed of sound c_s near the sound source as the average sound speed. Based on (14), when an ADCP is used to measure the current velocity in the horizontal direction with respect to the water depth, the measurements do not include any measurement errors resulting from the speed of sound varying with the water depth. In this profile, the speed of sound changes with depth and the bending of the sound waves caused by this variability cancel each other out.

In the above discussion, it was assumed that the forward and backward trajectories of the sound waves are the same.

On the other hand, when measuring the vertical component of the current velocity such as an upwelling current by using a vertical beam, the measurement errors based on (3) occur.

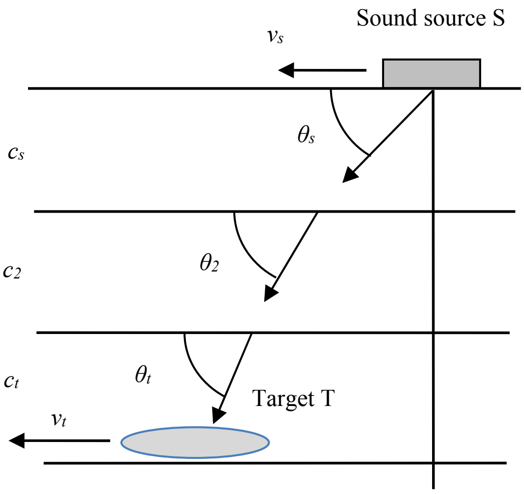

Fig. 3: Refraction of sound waves in a stratified structure.

6. CONCLUSION

The measurement errors caused by the assumption in the conventional Doppler shift equation that the speed of sound is constant in the sea were derived using the previously derived improved Doppler shift equation, which considers changes in the speed of sound along the path of propagation. When the speed of sound near the target differs from that near the sound source, the relative error of the target moving speed is expected to be of the same order of magnitude as the ratio of the speed of sound near the target to that near the sound source.

The improved Doppler shift equation indicates that the measured Doppler shift includes information about the speed of sound near the target. It is expected that the results of the present study may enable the remote measurement of the speed of sound near the target.

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