MODELLING OF AN ECHO-SIGNAL FROM A TARGET IN A WAVEGUIDE WITH A POSITIVE SOUND SPEED GRADIENT

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Abstract: *Target echo-strength measurements and modelling are usually made in a perfect environment where the sound speed of the surrounding medium is constant. In addition it is supposed that the object under study is in a free space. These ideal conditions never happen in reality. Most of the time the target to be detected is submerged in a waveguide where the sound speed is a function of depth. In present paper we study how both these additional conditions modify the acoustic scattering of a spherical target. To simplify the approach, we have considered a medium model where the scatterer, as well as the sound source/receiver, are located in a homogeneous water layer. Below this layer the sound speed increases with depth and the reflection coefficient squired changes linearly. For calculation the echo-signal in the frequency domain we have followed the Hackman and Sammelmann's general approach. The arising scattering coefficients of the sphere were evaluated with the use of the normal mode method. The amount of normal modes forming backscattered field is determined by the given directivity of the source. The emitted signal is a pulse with a cosine envelope and a pulse duration equal to 0.01 s. Computational results are obtained in a wide frequency range 70-90 kHz, distances between the source/receiver and a target from 500 m up to 1 km. A target is assumed to the acoustically rigid with a radius of 0.3-0.5 m. The effect on a target echo-strength of the ray paths which contribute to the scattering process as well as the surface wave the of the Franz type has been studied.*

Keywords: *Scattering of acoustic waves, echo-signal, spherical scatterer, normal modes of the waveguide, a waveguide with a positive sound speed gradient, pulse with a cosine envelope, surface waves of the Franz type.*

1. INTRODUCTION

In this paper we will study the medium model, where the spherical scatterer of a radius *a* as well as the source/receiver are located in a water layer with a constant sound speed c_0= *1461.45 m/s.* The positive *z*-axis is directed vertically upwards. The homogeneous layer is bounded by planes *z=d* and *z=-H*. When *z<-H* the sound speed *c(z)* increases with depth and the reflection coefficient squared $n^2(z)$ changes linearly

$$n^2(z) = 1 + \gamma(z + H), \quad z < -H, \gamma > 0. \tag{1}$$

It is assumed that *d+H=40* m. The sea depth is 200m and its bottom is acoustically rigid. The parameter γ in Eq.(1) is equal to $2.4633.10^{-5}\ m^{-1}$. This corresponds to a typical gradient $0.018\ s^{-1}$ of the sound speed in the Barentz sea. The frequency band of the point source placed at the point *M* is 70-90 kHz. The receiver is located at the point *M* as well. The radiater is assumed to be directed with the angular width equal to 4.5^{o} at the frequency of *80 kHz*. The Cartesian and spherical coordinates of the point *M* are *(0,y,z)* and *(r,θ,0)*, respectively; *y>0*. It is assumed that *0.3≤a≤0.5m*, *500≤r≤1000m*, *d=39m, a<z<d*. In this case, at the considered angular width of the source, the target is not illuminated by rays reflected from the upper boundary *z=d*. This allows to consider the upper waveguide boundary as pressure release, even if the presence of the homogeneous layer *-H≤z≤d* corresponds to the ice cover.

If z<-H, rays propagate along arcs and at *500≤r≤1000m* , these rays do not reach the bottom. This makes it possible, calculating the reflection coefficient from the interface *z=-H*, the use of the medium model with an infinite water half-space, where the sound speed changes in accordance with the law (1).

2. THEORY

In the frequency domain, solving the scattering problem formulated above, we will follow the Hackman and Sammelmann's general approach [1], where the acoustic potential of the backscattered field from a target is represented in the form

$$\Phi(\omega) = -\frac{i}{k_0}\sum_{l=0}^{\infty} T_l \sum_{m=0}^{l} A_{ml}(\bar{r}) C_{ml}(\bar{r}). \tag{2}$$

In (2) $k_0 = \omega / c_0$ is the wave number in the water layer *-H≤z≤d* , T_l are elements of the free-field T –matrix. For the acoustically rigid sphere

$$T_l = -j_l'(k_0 a) / h_l^{(1)\prime}(k_0 a). \tag{3}$$

In (3) $h_l^{(1)}(x)$ and $j_l(x)$ are the spherical functions. The coefficients $A_{ml}(\bar{r})$ in (2) are called the scattering coefficients of a sphere. They are of the form

$$A_{ml}(\bar{r}) = i^{l-m+1}\sqrt{\frac{\varepsilon_m}{2\pi}}\int_0^{\infty}\frac{qdq}{h} J_m(qy)\frac{\Pi_l^m\left(\frac{h}{k_0}\right)\left[e^{ih(z-d)} + Ue^{-ih(z-d)}\right]}{1 - UV\exp[2ih(d+H)]}\left[(-1)^{l+m}e^{ihd} + Ve^{ih(2H+d)}\right]. \tag{4}$$

In (4) $\varepsilon_0 = 1, \varepsilon_m = 2$ for $m \geq 1$, J_m is the m-th order Bessel function; q and $h = h(q) = \sqrt{k_0^2 - q^2}$ - are the horizontal and vertical components of the incident wave vector in a water layer $-H \leq z \leq d$, $\Pi_l^m(x)$ are the normalized associated Legendre functions, *U=-1* and *V(q)* are the reflection coefficients from the upper interface *z=d* and the interface *z=-H*, respectively,

$$V(q) = -\frac{Ai'(t_0) - i\sqrt{t_0}Ai(t_0)}{Ai'(t_0) + i\sqrt{t_0}Ai(t_0)}, \qquad (5)$$

where $Ai(x)$ is the Airy function,

$$-t_0 = \delta^2 k_0^2 \cos^2\theta_0, \quad \delta = \left(\gamma k_0^2\right)^{-1/3}. \qquad (6)$$

In (6) θ_0 is the incident angle of a plane wave from a homogeneouse layer $-H \leq z \leq d$ to the interface *z=-H*, $\gamma = 2.4633 \cdot 10^{-5} m^{-1}$. In the single-scatter approximation which is used below $C_{ml}(\bar{r}) = A_{ml}(\bar{r})$.

The truncation level $l_{\max}$ in (2) is set by the rule suggested by Kargl and Marston [2]

$$\max l = l_{\max} = \left[k_0 a + 4.05(k_0 a)^{1/3}\right] + 3, \qquad (7)$$

where *[x]* is the integer part of *x*. For *a=0.3 m* and *f=80 kHz* Eq. (7) gives $l_{\max} = 139$. Thus computing the backscattered field (2), it will be necessary to sum up more than 7500 summands. At frequencies and distances of interest, the integrands of integrals (4) are rapidly oscillating and slowly decreasing, that makes the straightforward calculations of the scattering coefficients (4) extremely time consuming. To speed up the calculation of integrals (4), we will calculate them by using the normal mode method.

The dispersion equation for finding the eigenvalues ξ_j can be written in the form

$$tg\left[k_0 \mu_0 (d + H)\right] = \frac{\sqrt{-t_0}Ai(t_0)}{Ai'(t_0)}, \qquad (8)$$

where $\mu_0 = \sqrt{1-\xi^2}, q = k_0\xi, \xi = \cos\theta_0$ If the radiation takes place inside a cone, having the angular width equal to α_a it is necessary to take into account only the propagating normal modes with ξ_j satisfying the inequality $\xi_j \geq \cos(\alpha_a/2)$.

3. COMPUTATIONAL RESULTS

To study the dependence of the backscattered field on frequency, let us consider the form function

$$F(f) = \frac{2}{a}\left|\Phi(f)/\Phi_{inc}(f)\right| = \frac{8\pi r^2}{a}\left|\Phi(f)\right|, \qquad (9)$$

where the acoustical potential Φ is given by Eq.(2), $\Phi_{Inc} = \exp(ik_0 r)/(4\pi r)$ is the potential of an incident wave at the origin of the coordinate when there is no scatterer.

We will compare the backscattered field of a spherical target in a waveguide and in a free water space, where

$$\Phi^{(f)}(f) = \frac{ik_0}{4\pi} \sum_{l=0}^{\infty} (2l+1) T_l \left[h_l^{(1)}(k_0 r) \right]^2 . \quad (10)$$

Figure 1 shows the dependence of the form function $F^{(f)}$ on $k_0 a$ at *r=500m* (see (10)). In the case of a free water space, the echo-signal consists of two interfering components: the geometrical-optical reflection and the circumferential surface wave of the Franz type.

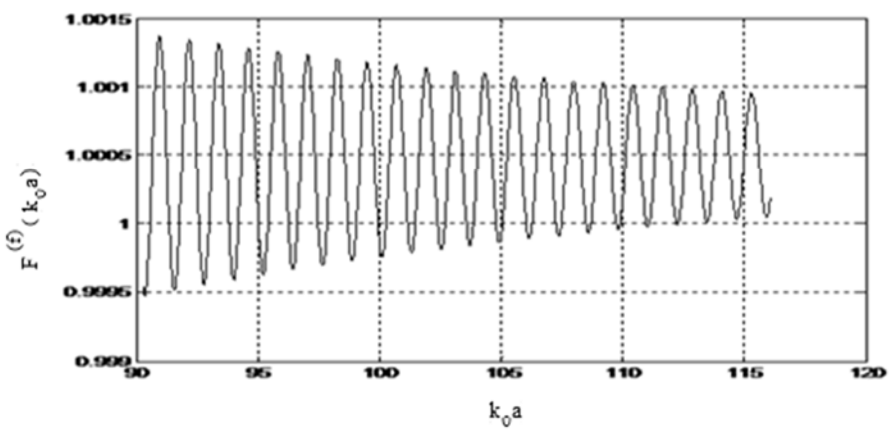

Fig.1: Backscattered form function from a scatterer in a free water at r=500m.

Difference in propagation times of these two waves at the observation point *M* is equal to

$$\Delta\tau = \left[2\sqrt{r^2 - a^2} + \pi a - 2(r-a) \right] / c_0 \approx a(\pi + 2 - a/r)/c_0 .$$

This gives the period of oscillations, expressed via $k_0 a$, equal $\Delta_{k_0 a} f = 2\pi a/(c_0 \Delta\tau)$. For *r=500m* and *a=0.3m* the last formula gives *1.2217*. In Fig.1 the period of oscillations is equal to *1.2084*.

Figure 2 shows the dependence of the form function (9) on $k_0 a$. Its shape demonstrates the interference of the geometrical-optical reflection with the wave reflected from the interface *z=-H* and with the circumferential surface waves of the Franz type.

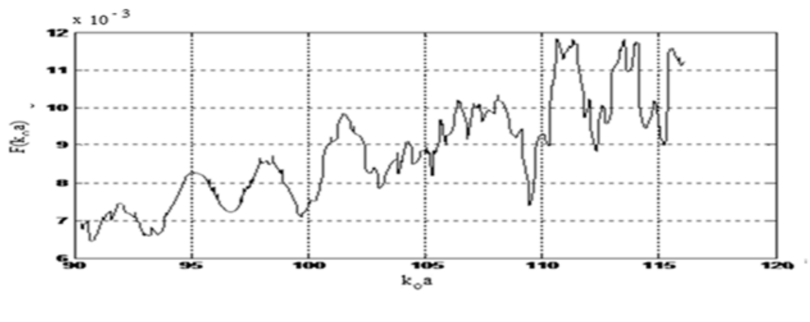

Fig. 2: Backscattered form function for a scatterer in a waveguide with the sound speed increasing with depth; r=500m,a=0.3m.

Let the source radiates a signal with a cosine envelope and duration $2\Delta t = 10\ ms$:

$$Y(t) = \begin{cases} 0, & t < -\Delta t, \\ \frac{1}{2}\left[1 + \cos\frac{\pi t}{\Delta t}\right] e^{-i\omega_c t}, & -\Delta t < t < \Delta t, \\ 0, & t > \Delta t, \end{cases}$$

where $\omega_c = 16\pi \cdot 10^4 s^{-1}$. The spectral density of this signal is

$$S(\omega) = \int_{-\Delta t}^{\Delta t} Y(t)\exp(i\omega t)dt = \frac{\pi^2 \sin(\Delta t(\omega - \omega_c))}{\left[\pi^2 - (\Delta t)^2(\omega - \omega_c)^2\right](\omega - \omega_c)}.$$

In the time domain we will compare signals, received at the point M in the free water

$$\hat{\Phi}^{(f)}(t) = \frac{1}{2\pi}\mathrm{Re}\left\{\int_{\omega_c - \Delta_\omega}^{\omega_c + \Delta_\omega} \Phi^{(f)}(\omega) S(\omega)\exp(-i\omega t)d\omega\right\},$$

where $\Delta_\omega = 2\pi \cdot 10^4\ s^{-1}$, $\Phi^{(f)}$ is given by Eq.(10), and in the case when the scatterer is located in the waveguide with the sound speed increasing with depth

$$\hat{\Phi}(t) = \frac{1}{2\pi}\mathrm{Re}\left\{\int_{\omega_c - \Delta_\omega}^{\omega_c + \Delta_\omega} \Phi(\omega) S(\omega)\exp(-i\omega t)d\omega\right\}.$$

In a free water space the arrival time of the geometro0opnical echo is equal to 0.6849 s and the arrival time of the first Franz wave is $\left(2\sqrt{r^2 - a^2} + \pi a\right)/c_0 \approx 0.6849s$. Thus, the Franz wave arrives at the observation point M in 1ms after the geometrical-optical echo interfering with it. The ratio of amplitudes of these two waves is more than 40. As a result, the pulse scattered in a free water space differs from the radiated pulse negligibly (see Fig.3).

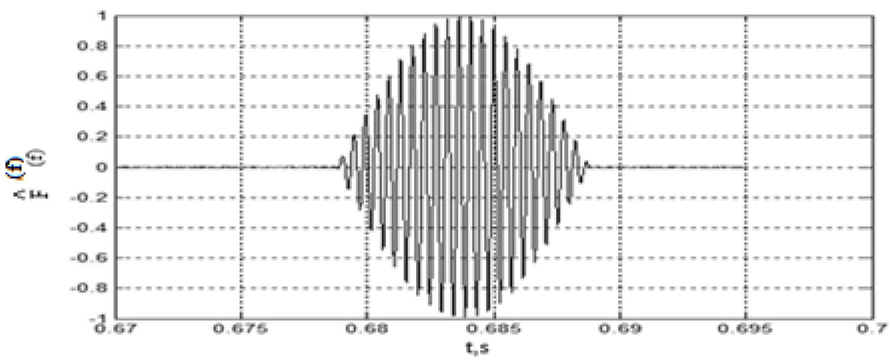

Fig.3. The normalized echo signal $\hat{F}^{(f)}(t) = \frac{8\pi r^2}{a}\hat{\Phi}^{(f)}(t)$ *reflected from the acoustically rigid sphere of a radius a = 0.3 m in a free water space.*

In the case of a waveguide, the scattering geometry is supplemented with a ray reflected from the interface *z=-H* and with a ray propagating partly in the inhomogeneous water layer z<-*H*. The propagation times of three signals corresponding to a ray reflected from the interface *z=-H* differ from the propagation time of the geometrical-optical echo for less than *1ms*. As a result, a strong interference is observed. The considered angular width of the source is 4.5º. In this case the ray, partly propagating in the inhomogeneous layer, reflecting from the sphere and going back to the point *M* along the same way exists only if *r>6 km*. Thus, the corresponding rays will not illuminate the target (see Fig.4).

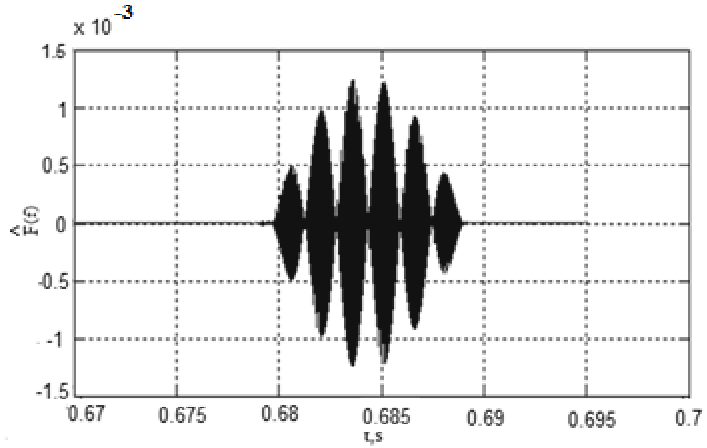

Fig.4: The normalized echo signal $\hat{F}(t)=\frac{8\pi r^2}{a}\hat{\Phi}(t)$ *reflected from the acoustically rigid sphere of a radius a = 0.3 m submerged in a waveguide.*

4. ACKNOWLEDGEMENTS

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