UXO IDENTIFICATION USING A MATRIX PENCIL APPROACH

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Abstract: *The detection and identification of Unexploded Ordnance (UXO) at former military sites using sonar presents significant challenges which can be attributed to the presence of clutter objects of similar size and shape to targets of interest. Remediation of large UXO sites requires computing features that provide Automatic Target Recognition (ATR) algorithms with the ability to reliably discriminate between man-made and clutter objects based on their acoustic returns. This paper presents the use of the Matrix Pencil (MP) algorithm to enable the identification of underwater targets potentially observed from multiple aspect angles. The Matrix Pencil algorithm models the signal reflected from a target as linear combination of exponential modes. Importantly, using the Matrix Pencil approach requires only a single data snapshot to estimate the modes, i.e. we do not require an estimate of any second order statistics. The modes are determined as the generalized eigenvalues of a pair of matrices created using a single-time sample shift operator. The modes estimate the resonant frequencies (the imaginary component) and the attenuation (the real component) of the ensonified objects. The amplitudes of the modescan can then be obtained using a least-squares estimate. The Matrix Pencil approach is tested using the well-known PONDEX dataset, which consists of calibrated scattering data from different aspects of several targets, including UXO shells, collected using a rail system. Results show that the Matrix Pencil approach, is able to reliably classify targets.*

Keywords: *UXO Identification, Matrix Pencil, Nearest Neighbour, Classification, Automatic Target Recognition*

1. INTRODUCTION

The automatic recognition of underwater unexploded ordinance (UXO), specifically, distinguishing UXO from clutter with similar size and shape remains an open, but important, problem. In our context, automatic target recognition (ATR) requires the characterization of objects of interest via a training phase, following by matching a measurement to these characteristics [1]. In this regard, ATR requires two key steps: creating a library of useful characteristics in the training phase, i.e., extracting key target information in the training phase, and the use of an effective classification approach in the recognition phase. This paper focuses on the first of these two steps, specifically developing the use of the Matrix Pencil algorithm [2][3] to characterize targets of interest. We then use the well-known nearest neighbour algorithm [4] in the classification step to match a measurement, also processed by the Matrix Pencil algorithm, to the extracted target characteristics.

The Matrix Pencil algorithm is an example of model-based parameter estimation. The approach models a sampled time domain signal as a linear sum of complex exponential modes; the algorithm them finds the parameters that define the linear sum: the mode exponents and the related mode amplitudes. While a sum-of-exponentials is a well-established model, Matrix Pencil differs from other techniques in not requiring any statistical information; a single snapshot of collected data is adequate to execute the algorithm. The approach has been used in multiple applications such as indoor localization [5] and compensation for mutual coupling in adaptive processing [6].

The key to the Matrix Pencil approach is that it captures the essential behaviour of a target within a few numbers – importantly, these numbers have a physical interpretation as well. The real component of the modes indicate the mode attenuation – we would expect a hollow shell to have modes with a low mode attenuation to model the “ringing”. On the other hand, the imaginary component of the mode exponent relates to the target resonances. Finally, while the absolute value of the amplitudes depends on the intensity of the incident waves, the *relative* values of the amplitudes indicate the relative strengths of the target modes (assuming that the incident wave is broadband enough).

In this paper, the Matrix Pencil approach is applied to the PondEx dataset [7]. The dataset comprises time domain measurements of the signals reflected off several different types of targets placed in several different orientations .The data was collected using a rail system, providing several cross-range samples of the target response in addition to the multiple target orientations. The targets include rocks, UXO aluminium shells, pipes and solid cylinders. Our examples here will largely focus on the aluminium targets.

This paper is organized as follows: Section 2 develops the theory of the Matrix Pencil algorithm. Section 3 presents the characterization and testing methodology. Section 4 presents the results based on our preliminary numerical testing using the nearest neighbour classification algorithm. Finally, Section 5 wraps up the paper.

2. THE MATRIX PENCIL ALGORITHM

In the Matrix Pencil approach, a time domain signal, $x(t)$, is written as a linear sum of complex exponentials

$$x(t) = \sum_{m=1}^{M} A_m e^{\zeta_m t}; \quad \zeta_m = \alpha_m + j\beta_m, \tag{1}$$

where $\zeta_m, m = 1, \ldots, M$, represent the M complex modes of the system. The real component of the mode, α_m, denotes the mode attenuation, while, the imaginary component, β_m denotes the mode frequency. The signal is sampled N times with a sampling period of Δt. Let $z_m = e^{\zeta_m \Delta t}$; we get

$$x[n] = x(n\Delta t) = \sum_{m=1}^{M} A_m z_m^n; \quad n = 0, 1, \ldots, N-1, \tag{2}$$

The goal of the Matrix Pencil approach is to estimate $z_m, m = 1, \ldots, M$, the poles, and $A_m, m = 1, \ldots, M$ the mode amplitudes. We accomplish this as follows: we first choose the *pencil parameter* L ($\simeq N/2$) and form the $(N-L) \times (L+1)$ matrix $\mathbf{X}$:

$$\mathbf{X} = \begin{bmatrix} x[0] & x[1] & \cdots & x[L] \\ x[1] & x[2] & \cdots & x[L+1] \\ \vdots & \vdots & \ddots & \vdots \\ x[N-L-1] & x[N-L] & \cdots & x[N-1] \end{bmatrix}. \tag{3}$$

Based on this matrix, we can define two $(N-L) \times L$ submatrices $\mathbf{X}_0$ and $\mathbf{X}_1$ representing the first and last L columns of $\mathbf{X}$ respectively. Note that there is a single time-shift between the two matrices. Importantly, these two matrices can be written as

$$\mathbf{X}_0 = \mathbf{Z}_1 \mathbf{A}\, \mathbf{Z}_2, \tag{4}$$

$$\mathbf{X}_1 = \mathbf{Z}_1 \mathbf{A}\, \mathbf{Z}_0 \mathbf{Z}_2, \tag{5}$$

where

$$\mathbf{Z}_1 = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ z_1 & z_2 & \cdots & z_M \\ \vdots & \vdots & \ddots & \vdots \\ z_1^{N-L+1} & z_2^{N-L+1} & \cdots & z_M^{N-L+1} \end{bmatrix}, \quad \mathbf{Z}_2 = \begin{bmatrix} 1 & z_1 & \cdots & z_1^{L-1} \\ 1 & z_2 & \cdots & z_2^{L-1} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & z_M & \cdots & z_M^{L-1} \end{bmatrix}, \tag{6}$$

$$\mathbf{Z}_0 = \mathrm{diag}(z_1, z_2, \ldots, z_M) \tag{7}$$

$$\mathbf{A} = \mathrm{diag}(A_1, A_2, \ldots, A_M) \tag{8}$$

where diag( ) denotes a diagonal matrix with the corresponding entries on the diagonal. Based on (7) and (8), our goal is to estimate the matrices $\mathbf{Z}_0$ and $\mathbf{A}$.

Consider the following formulation, called a “matrix pencil”

$$\mathbf{X}_0 - \lambda \mathbf{X}_1 = \mathbf{Z}_1 \mathbf{A} [\mathbf{Z}_0 - \lambda \mathbf{I}] \mathbf{Z}_2. \tag{9}$$

Choosing $\lambda = z_m$, for some m, reduces the rank of the pencil by one. The estimates for z_m are, therefore, the *generalized eigenvalues* of the matrix pair $[\mathbf{X}_1, \mathbf{X}_0]$. This approach allows for the unambiguous estimation of the poles if $M \leq L \leq N - M$.

Given the poles, the attenuation factor, $\alpha_m = \Re\{\log(z_m)\}/\Delta t$ and $\beta_m = \Im\{\log(z_m)/\Delta t)\}$ (here $\Re\{\ \}$ and $\Im\{\ \}$ denote the real and imaginary parts of a complex number). The conversion to continuous time parameters assumes that the Nyquist sampling rate constraint has been met. Finally, the amplitudes can be estimated using a least squares solution of

$$\begin{bmatrix} x[0] \\ x[1] \\ \vdots \\ x[N-1] \end{bmatrix} = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ z_1 & z_2 & \cdots & z_M \\ \vdots & \vdots & \ddots & \vdots \\ z_1^{N-1} & z_2^{N-1} & \cdots & z_M^{N-1} \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \\ \vdots \\ A_M \end{bmatrix}. \tag{10}$$

The development so far has assumed the number of exponents in the model, M, is known. In practice, this could be estimated using the distribution of singular values of the matrix **X** [2]. However, here we use a fixed value (found largely through trial and error).

3. USING MATRIX PENCIL WITH PONDEX DATA

As the previous section has shown, the Matrix Pencil algorithm models a single time domain signal as a linear sum of complex exponents. For every object and orientation, measurements were taken at 800 cross-ranges that are 2.5 cm apart covering 20 meters. In all, each target is measured in 9 orientations (0°, ±20°, ±40°, ±60° and ±80° with respect to vertical). The transmitted pulse was a 6 ms chirp from 1 to 30 kHz and the received pulses were match filtered and sampled at with period $\Delta t = 1 \times 10^{-5}$ for 20 ms resulting in 2000 samples. As a result, each orientation is associated with an 800 by 2000 matrix where each row is a time-domain signal occurring at a cross-range

Since the matched filtering creates a "two-sided" time-compressed signal, we model the tail starting with the peak and processing N samples. This choice covers the signal for all targets. We use the Matrix Pencil approach to model the signal at each cross-range position and each target orientation using M modes. Since the measured time domain signal is real, the modes form conjugate pairs, i.e., there are $M/2$ unique modes per signal (resulting in each signal represented by $3M/2$ numbers ($M/2$ amplitudes, attenuations and resonant frequencies). These values can then be used as the basis for the classification process). In the classifier we set $N = 512$ and $L = 255$. The peak signal value is normalized to unity.

Based on the given data, there are several ways to train and test a classifier. In this paper we use a stringent approach: the training set was obtained using data from the 20°, 40 °, 60 °, and 80 ° orientations, while the testing used data from the -20°, -40 °, -60 °, and -80 ° degree orientations. Using this approach there are roughly 1000 vectors for training (to characterize a target) and a similar number for testing.

Classification is attempted using a single cross-range measurements. A subset of representative cross-ranges was obtained from each object and orientation. This was done by choosing the cross-range with the highest energy time-domain signal and restricting the cross-ranges to ensure that the measured signals had energy within 8dB of the peak.

We use the principal components analysis (PCA) approach as a baseline. The PCA starts with the raw time-domain data (covering the training set) in the form of a matrix and uses a singular value decomposition, retaining the M singular vectors, corresponding to the M largest singular values. These M singular vectors form the projection matrix: the training measurements are projected onto the associated subspace, providing the feature vectors.

4. NUMERICAL RESULTS

4.1 MATRIX PENCIL

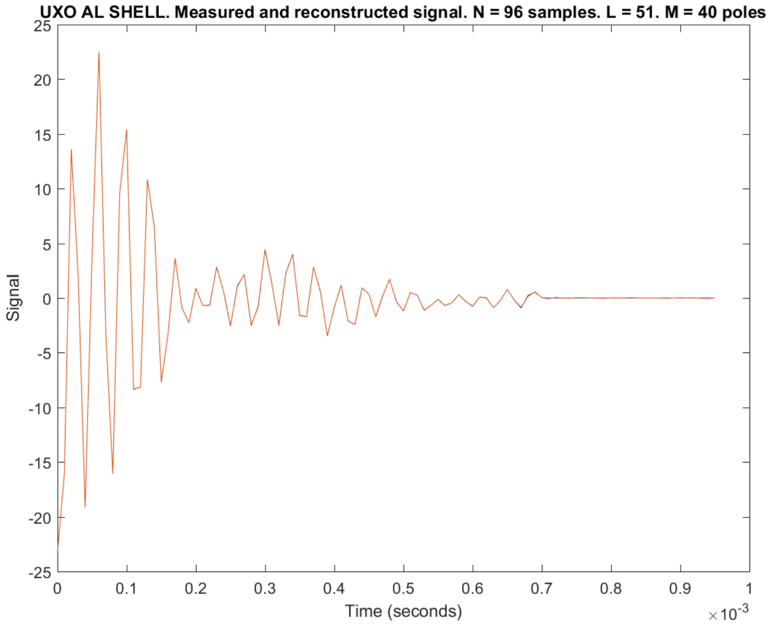

Fig. 1 : Comparing the measured and reconstructed signals using the Matrix Pencil model

Fig.1 illustrates the efficacy of the Matrix Pencil approach. To generate a sample result, we use the UXO Aluminium shell (UXO_AL_SHELL) at broadside and 0° orientation. In this plot, we use $N = 96$ time samples and set $L = 51$ and $M = 40$. The plot overlays the original measured signal and the signal estimated obtained using the Matrix Pencil estimate of the amplitudes and poles in conjunction with the model in Eqn. (2). The two signals overlap almost exactly. As is clear, the Matrix Pencil approach is remarkably accurate in its reconstruction of the signal.

Our second sample result illustrates that the parameters obtained using the Matrix Pencil model has a physical interpretation. Fig. 2 plots the frequency spectrum of the measured signal and overlays (by scaling the pole magnitudes so the comparison is easy to visualize) the mode frequencies and amplitudes. As can be seen, the locations of the mode frequencies (coupled with their magnitudes) corresponds well to the frequency spectrum.

Fig. 3 presents visual evidence of using the Matrix Pencil approach to discriminate between types of objects. The figure plots the minimum value of the attenuation (the min. α_m) a function of cross-range positions (centered at broadside) for 4 target types: two different rocks, the UXO shell, and an aluminium pipe. Intuitively, we would expect the pipe to have modes with lower excitation (due to 'ringing'). The plot clearly shows that this is true with the mode with minimum attenuation far separated from the other three.

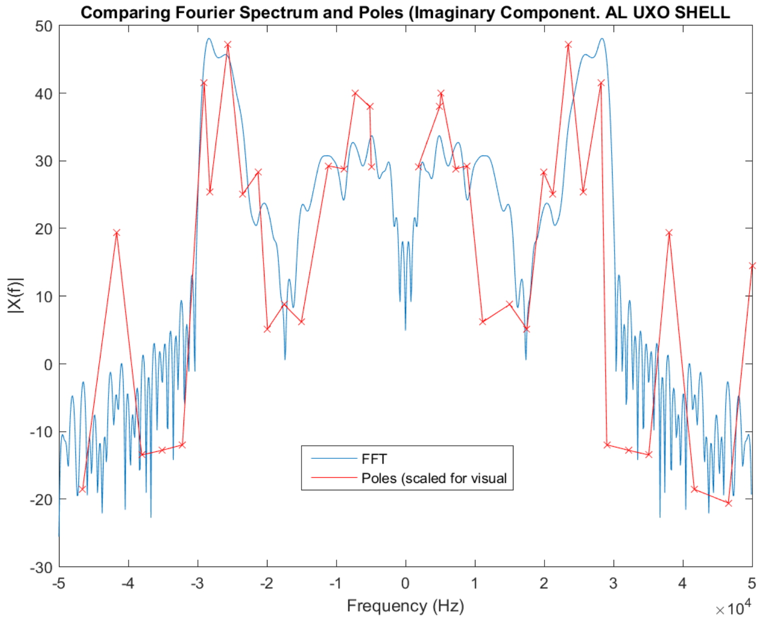

Figure 2: Comparing Fourier Spectrum and Mode Frequencies.

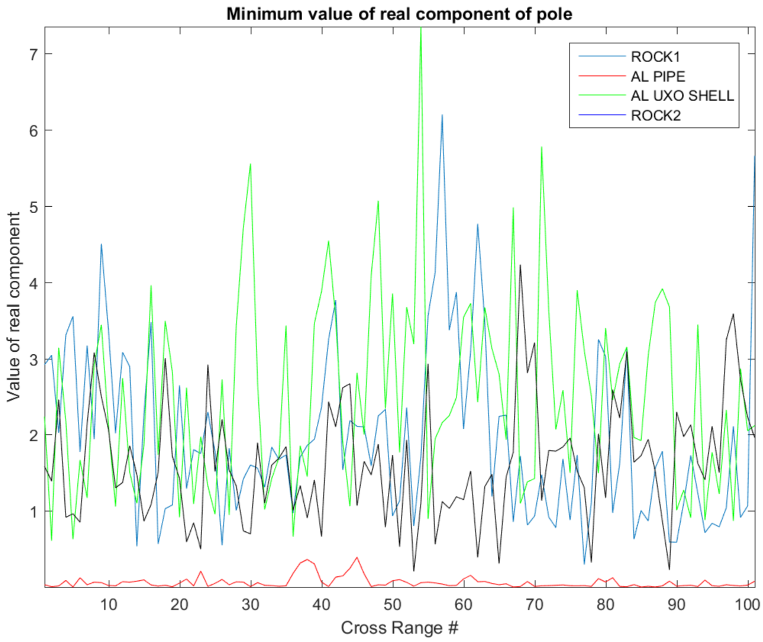

Figure 3: The min attenuation for several targets and cross-range samples.

4.2 CLASSIFICATION TESTING

The training data is used to generate corresponding to the cross-ranges and training orientations. In the most part, we used $M = 40$, resulting in 20 unique modes. Classification is based on the real component (the attenuations), the imaginary components (the resonant frequencies) or the two appended. The first two approaches provide a length-20 feature vector for each cross-range position and orientation; the third approach results in a length-40 feature vector for each cross-range position and orientation.

Classification is based on the “K-nearest neighbour” or KNN algorithm – given a test vector, we first extract its features using the Matrix Pencil approach. We then find the K nearest feature vectors from the training data (corresponding to all the targets in the classification). The choice is the target class that has the maximum representation amongst the K nearest neighbours (if $K = 1$ then we just choose the target corresponding to the closest feature vector from the training set).

Aluminium Pipe and Shell: The first experiment classifies between the aluminium pipe and shell. Here we used $K = 10$ and an L_1-distance metric. Table 1 lists the classification accuracy achieved using $M = 40$ (effective dimension of 20) with the Matrix Pencil approach and compares it the accuracy of the PCA algorithm. As mentioned before, there are approximately 1000 test signals in the test orientations chosen.

Feature Vector	Dimensionality	Test Accuracy
Real part of poles	20-D	82.9%
Real and imaginary parts of poles appended and standardized	40-D	83.5%
PCA on time-domain half-signal retaining 20 components	20-D	71.0%

Table 1: Classification Accuracy Between Aluminium Pipe and Aluminium Shell

As listed in the table, using just the attenuations results in an 82.9% classification accuracy on the test set (as compared to a 71.0% classification accuracy using PCA). Appending the real and imaginary parts of the pole vectors to produce 40-dimensional feature vectors and standardizing the data yielded a test accuracy of 83.5% for the same model type. Given that the two objects are of a similar size (implying resonant frequencies) incorporating the imaginary parts of the poles yields only a marginal improvement.

Aluminium Pipe, Shell and UXO : We repeated the experiment with $K = 1$ classifying between three types of targets. Using the real part of the feature vectors (attenuations) yields a classification accuracy of 79.7% on the test set for this model type. Furthermore, the classification accuracy for distinguishing between the solid aluminium cylinder and pipe was found to be 72.2% suggesting that they were now being occasionally misclassified as shells. Using the imaginary parts of the pole vectors yields a classification accuracy of 73.4% and combining the real and imaginary parts by appending the vectors and standardizing yields an accuracy 82.0%. Given the difference in size difference between the shell and the pipe and cylinder, it is expected that the imaginary component would aid in discrimination. For comparison, using PCA on the time-domain half-signals yielded a test set classification accuracy of only 69.6% for 20 components and 65.2% for 40 components.

Feature Vector	Dimensionality	Test Accuracy
Real part of poles	20-D	79.7%

Imaginary part of poles	20-D	73.4%
Real and imaginary parts of poles appended and standardized	40-D	82.0%
PCA on time-domain half-signals keeping 20 components	20-D	69.6%
PCA on time-domain half-signals keeping 40 components	40-D	65.2%

Table 2: Three Way Classification Accuracy between Aluminium Shell, Pipe and UXO

5. CONCLUSIONS

In this paper we presented a new approach to extract the key features of a target with the aim of distinguishing between various types of targets; the approach is based on the Matrix Pencil algorithm. The algorithm models a time domain signal as a linear sum of complex modes. The mode attenuations and frequencies provide the characteristics of the target. We presented numerical results based on our preliminary testing of classification based on the Matrix Pencil approach. The approach is able to consistently provide classification accuracy of greater than 80% amongst the aluminium targets in the PondEx data set (and an improvement of over 10% as compared to the PCA approach). However, these are just preliminary results and significant testing remains to be done.

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