DISCRETE TIME TECHNIQUES FOR DIGITAL HYDROPHONE CALIBRATION

Jay Abela

Oceans Sonics, Ltd, 11 Lornevale Road, Great Village, Nova Scotia, B0M 1L0, Canada

Contact author: Jay Abel
Postal Address: Ocean Sonics, Ltd, Hill House, 11 Lornevale Road, Great Village, Nova Scotia, B0M 1L0, Canada
Fax: +1-902-655-3001
E-mail: jay.abel@oceansonics.com

Abstract: A digital hydrophone is an acoustic acquisition system comprised of a sensing element, amplifier, acquisition subsystem and means for communicating a digital representation of the received signal. The entire system is integrated into a single waterproof enclosure which becomes part of the acoustic environment. Since the operator does not have access to the analogue hydrophone output, calibration signal amplitudes must be determined from the digital output alone. The requirements for calibrating analogue hydrophones still apply, however additional challenges to be overcome include: Synchronization of the digital output signal to the projector and reference hydrophone waveforms; accurate determination of test tone amplitudes from data sampled at rates close to the Nyquist-Shannon limit; low-frequency amplitude errors caused by improper signal gating, and accommodating the waterproof enclosure, which in some cases may be too large for a standard test tank, and which may affect acoustic compliance. Sources of amplitude error as high as 1 dB at low frequencies and up to 6 dB for certain higher frequencies are identified. Specific recommendations are given to address the challenges identified.

Keywords: Digital Hydrophone, Calibration
1. CALIBRATION OF DIGITAL HYDROPHONES

Best practices for calibrating analogue hydrophones, including those with integrated preamplifiers, are well established\[1\]. Digital hydrophones, in contrast to their analogue counterparts, incorporate several functions into their waterproof enclosure, including amplification, signal conditioning, data acquisition, and in some cases even digital signal processing. Despite the obvious similarity between discrete-time sampled and continuous time systems, there are subtle differences which must be respected.

1.1. Digital Hydrophone Overview

There are important differences between digital hydrophones and their analogue counterparts:

- Sensor voltage and current nodes are inaccessible so calibration of the instrument must be made with respect to the digital output only.
- Larger housing due to incorporation of signal processing electronics. This may impose lower frequency limits on non-free-field measurements.
- Reciprocal operation is generally not supported, in which case a standard reference hydrophone will be required.

At the time of this writing, free-field calibration by comparison to a standard reference is the best available method for digital hydrophone calibration. Test signal amplitude must be determined from the output digital data stream alone, and the resulting sensitivity determined is thus a measure of the sensitivity of the discrete, quantized output of the instrument relative to the level of the applied continuous-time pressure stimulus.

1.2. Digital Sensitivity

A standard method of stating the sensitivity of digital hydrophones is needed. Some vendors continue to use units of dB re V / µPa, referring to the voltage sensitivity at a point in the signal chain after any amplification and analogue signal conditioning, at the point where the analogue signal is sampled. The relationship between signal voltage and digital counts is not implied. Units of dB re µPa\(^{-1}\) have also been proposed\[2\], though in the case of fixed point digital representations since it is unclear whether the unit value is a single count, full scale positive or some arbitrary count value.

Some laboratories resolve this ambiguity by stating the result in dB re count\(^2\) / µPa\(^2\). While this addresses the ambiguity present in reciprocal µPa, this can also result in significant confusion if an instrument is capable of multiple digital resolutions (for example, 16-bit and 24-bit).

Regardless of the method used, the details of the digital encoding must also be made known, including the format and resolution of the result (fixed or floating point), most positive and least negative digital values returned (e.g., \([-2^{23}, 2^{23}-1\], \([-1.0, 1.0\]), etc.) and in the case of sensitivity specified in dB re V\(^2\) / µPa\(^2\), a suitable gain factor (e.g., 6.0 V / \(2^{24}\)) for conversion from digital counts to input-referred volts.
1.3. Equipment Setup

The observations and recommendations in this work are derived from efforts to produce a commercial quality calibration facility for routine calibration of hydrophones against a standard reference unit. Because no standard exists for calibrating digital hydrophones, we were required to bootstrap the calibration for our reference unit by separately calibrating the element and digital signal chain. The uncertainty associated with acoustic interference from the equipment housing remains a significant concern.

![Fig. 1: Experimental Setup](image)

The experimental setup is shown in Fig. 1. The calibration tank is a 6000-litre fiberglass tank housed in a purpose-built outbuilding. A rubber mat helps to isolate the tank from building vibration. Behind the walls, ceiling, and floor, a wire screen mesh installed during construction to reduce electrical interference. Temperature is carefully maintained at 18 °C, chlorine concentration at 0.5 ppm, and pH is maintained between 6.5 and 7.0. Aside from occasional vacuuming with a siphon hose, the water in the tank is not otherwise circulated or filtered. As water evaporates it is replaced with tap water that has been held in settling tanks.

A digital projector is located on the fixed central post extending down from the ceiling, with the reference and unit under test suspended from a hub that can be rotated by 180 degrees. As shown in Fig. 1, the reverb-free distance is approximately 0.41 m. A pulley system lifts the equipment out of the water when required. The superstructure above the waterline is aluminium, and the support posts extending below are plastic. It has been suggested that glass fibre would provide superior support[3] and this is a planned upgrade.

1.4. Calibration Procedure

Two sets of measurements are made for each horizontal orientation of the hydrophone to be tested. In the first measurement, the reference is on the left-hand support with the unit under test on the right; in the second measurement, the positions of the instruments are swapped by rotating the frame (the projector does not rotate). The process assumes that projector directivity is constant but not negligible, that the test tone amplitudes may vary slightly between measurements, and that hydrophone and reference horizontal orientations is maintained when the frame is rotated.
Let $p_1$, $p_2$ be the projector amplitudes of the first and second measurements; let $D_A$ and $D_B$ be the relative directivity (gain) associated with the left and right fixture positions, and let $M_R$ and $M_H$ be the sensitivities of the reference and hydrophone, respectively. If $V_{HB1}$ is the amplitude recorded by the hydrophone in position B on measurement 1, $V_{HA2}$ is the amplitude recorded in position A on measurement 2, etc., so that

$$
M_R = \sqrt{\frac{V_{HB1} \cdot V_{HA2}}{V_{RA1} \cdot V_{RB2}}} \cdot \sqrt{\frac{(M_H \cdot D_B \cdot p_1)}{(M_R \cdot D_A \cdot p_1)}} \cdot \frac{(M_H \cdot D_A \cdot p_2)}{(M_R \cdot D_B \cdot p_2)} \cdot M_R = \sqrt{\frac{M_H^2}{M_R^2}} \cdot M_R = M_H^2
$$

In each measurement, the digital projector generates a series of 200 test tones organized as 5 series of 20 frequencies each, with each frequency repeated twice, once in phase and once in quadrature. The hydrophone and reference record each series in .wav format for later processing.

The projector, reference and unit under test are synchronized to a pulse-per-second (PPS) signal that has been pulse-width modulated to encode time and date information in a 50-bit record each minute. The hydrophone, reference and projector sample clocks, time and date are all locked to the PPS reference signal, so that each one-minute record is sample accurate to ±125 ns.

2. DISCRETE TIME TECHNIQUES FOR AMPLITUDE DISCOVERY

Only the reverb-free portion of each test tone after allowing for finite projector bandwidth and Q can be used as a repeatable, bandwidth limited test source[1]. In addition, there are additional constraints which must be observed when attempting to approximate continuous time amplitudes from sampled data.

2.1. Low Frequency Techniques

At low frequencies, projector Q, SNR and a limited number of full cycles all contribute sources of error. While not strictly arising from the sampled nature of the test signal, these effects are presented here as part of a comprehensive approach to the automated determination of test tone amplitudes.

Limited SNR (projector amplitude and limited bandwidth limit accuracy at lower frequencies) suggests that amplitudes determined using the entirety of valid sample data will be of higher accuracy than calculations based on one or two peak values. For this reason, peak and peak to peak methods are discouraged.

![Error dB vs Frequency](image)
Figure 2 shows the result of calculating the RMS level of a signal over a period of less than one cycle. Two extremes of phase alignment are plotted against the number of samples accumulated in the mean. Notice that the cumulative RMS total is equal to the cycle RMS only at half-cycle multiples. The uneven appearance of the graph is due to normally distributed noise added to both cycles to simulate the effect of 20dB SNR.

### 2.2. Orthogonal Test Signal Pair

If a pair of test tones are generated, as in the present system, such that the relative phase difference between them is 90 degrees, the instantaneous power sum (see Fig. 2) of the two signals is theoretically constant.

Consider two test tones with an orthogonal phase relationship, period $T$, and amplitude $A$ so that the mean squared amplitude is $A^2/2$. The absolute phase delay $\varphi$ is unimportant, provided it is constant. Uncorrelated noise amplitudes $\varepsilon_1$ and $\varepsilon_2$ are superimposed on the two tones. Now let $\varepsilon_3$ represent the variation in amplitude between the two tones. Computing the square of both tones, integrating over a single period, the sum of the results is twice the mean squared level plus the mean squared amplitudes of the three error sources assumed:

$$\frac{1}{T} \int_0^T [(A + \varepsilon_3) \cdot \cos \left( \frac{2\pi t}{T} + \varphi \right) + \varepsilon_1]^2 + [(A - \varepsilon_3) \cdot \sin \left( \frac{2\pi t}{T} + \varphi \right) + \varepsilon_2]^2 \, dt = A^2 + \varepsilon_1^2 + \varepsilon_2^2 + \varepsilon_3^2 \quad (2)$$

The various error sources $\varepsilon_1, \varepsilon_2, \varepsilon_3$, can be constants, orthogonal frequency components or uncorrelated noise sources since all these will, at least statistically, produce equal variances when integrated as shown.

We can recover the original tone amplitude by calculating the discrete-time Fourier series coefficients from the in-phase and quadrature test tones. This method is superior to the straightforward RMS calculation because only power in the test tone frequency range is considered. Referring to (3), let $k$ be the number of complete tone periods in $N$ samples, $k$ and $N$ integers, and with $I$ and $Q$ vectors containing the sampled waveforms in units of $\mu$Pa, then $L_P$ is the recovered sound pressure level.

$$L_P = 10 \log_{10} \left[ \frac{1}{N} \sum_{n=0}^{N-1} |I[n]e^{-j\frac{2\pi kn}{N}}|^2 + \frac{1}{N} \sum_{n=0}^{N-1} |Q[n]e^{-j\frac{2\pi kn}{N}}|^2 \right] \quad (3)$$

In addition to offering inherent rejection of DC offset, the above method also provides out of band rejection with a bandwidth of approximately $f_s/N$, where $f_s$ is the sample rate and $N$ is the number of samples over which the DFT is calculated, above.

### 2.3. High Frequency Considerations

At frequencies near half the sample rate, $f_s$, aliasing effects limit the maximum test tone frequency $f_t$ for which RMS calculations will give accurate results. The number of
samples required to achieve an accurate estimate of amplitude to within 1.0 dB is approximately:

\[ F_t < \left( \frac{1}{2} - \frac{1}{2.2N} \right) \times F_s \]  \hspace{1cm} (4)

When using two tones in quadrature, there is no restriction on the tone frequency that will give accurate results, including exactly half the sample rate (though the acquisition system probably employs an anti-aliasing filter which obviously must be respected).

It might be tempting to estimate frequency amplitudes from the observed peak or peak to peak values. When the test tone period is a multiple of the sample rate certain phase offsets will produce curious results. Despite appearances, the waveform in Fig 3a does not have a DC offset, and Fig 3b has an amplitude of exactly ±A.

Fig.3: Aliasing at frequencies well below f_s/2: a) Left, f_s/3, b) Right, f_s/4

3. CONCLUSION

Accurate calibration of digital hydrophones requires the accurate determination of test tone amplitudes at frequencies very close to half the sample rate. The use of a pair of test tones with a relative phase separation of 90 degrees in conjunction with DFT post-processing provides an accurate estimate of the test tone level even in the presence of external interference, system noise and dc offset. Modifying the test tone frequencies so that \( k \) complete cycles of the test tone occupy exactly \( N \) samples, the \( N \) samples chosen to be within the projector Q-limited and reverb-free interval, will further reduce processing error.

The result of the calibration will yet fail to be meaningful without a standard method for characterising the output level. Until a standard exists, for whatever units are used to express the result, the specific dc transfer characteristic and digital sample resolution of the quantization stage must be specified.

REFERENCES

[1] **BS EN 60565:2007**, *Underwater acoustics – Hydrophones – Calibration in the frequency range 0.01 Hz to 1 MHz*