AN IMPROVED SQUINT WIDEBAND SYNTHETIC APERTURE SONAR IMAGING ωk ALGORITHM

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Abstract: *The ωk algorithm is a true broadband, wide-beam and high-squint imaging algorithm. In the treatment of squint problem, the two-dimensional wavenumber domain tilt caused by the non-zero Doppler center, made the azimuth wave number processing more complex, which reduces the data utilization efficiency and image quality. Aiming at the problem of broadband squint SAS data imaging, we propose a method for Doppler centroid correction by simply multiplying the phase term in the range wavenumber domain and azimuth space domain, and then constructs an equivalent SAS side-looking geometry model by constructing a new wavenumber domain coordinate system. Then, interpolation is performed in the distance wavenumber domain and the azimuth wavenumber domain respectively, and the two-dimensional wavenumber domain data under broadband squint condition is transformed into equivalent side-looking data, and ultimately focus the image using the classic ωk algorithm. The algorithm can realize the image focusing and avoid the large amount of zero-padding and data movement in the algorithm of distance migration under squint condition, and improve the data utilization and processing efficiency. Point target simulation data processing verifies the effectiveness and practicability of the proposed algorithm.*

Keywords: *SAS Imaging; Wide-band; Squint; Wavenumber Domain Coordinate Rotate; Modified ωk Algorithm*

1. INTRODUCTION

At present, some typical SAS [1] mostly work in broadband or ultra-wideband conditions. Multi-receiving array SAS is generally used in order to overcome the single-array SAS which cannot simultaneously obtain high azimuth resolution and high mapping rate problem, Firstly, the echo data of multi-receiving array are preprocessed and then imaged by single-array SAS rapid imaging algorithm[2].The ωk algorithm(ωkA) is a true broadband, wide beam and high squint imaging algorithm. In squint conditions, additional processing is required for Doppler frequency beyond the azimuth bandwidth [3] , generally the azimuth data being zero-padded to extend the azimuth spectrum width, which simplifies azimuth spectrum folding and Doppler centroid change[4], but it will increase the computational complexity.

In view of the superior ability of the ωkA to deal with squint data, this paper presents an improved ωkA for striped broadband squint SAS imaging. Firstly, the constraint relations of two-dimensional wavenumber spectrum are analyzed in broadband squint mode. Then the Doppler center frequency is corrected to zero frequency in the distance wavenumber and the azimuth space domain, and the analytical form of the two-dimensional wavenumber spectrum of the SAS echo signal after zeroing is deduced, the azimuthal compression factor introduced by the resampling of the azimuth spectrum and Taylor series expansion approximation of two-dimensional spectrum in [5] is removed. Furthermore the concept of rotating ωkA used in SPOTLIGHT SAR is introduced, by rotating the coordinate axes of the two-dimensional wavenumber domain, the equivalent side-looking geometry SAS model was constructed, the method of transforming the azimuthal coordinate axis in the rotating ωkA of SPOTLIGHT SAR is simplified by using Doppler zeroing. Lastly, the rotation and STOLT interpolation of the data in the new coordinate system are completed by interpolating the azimuth wavenumber and the range wavenumber respectively. Finally, two-dimensional inverse Fourier transform is used to realize image focusing. By using the improved RMA to process point target simulation data, the validity and practicability of the proposed method are verified.

2. IMAGING GEOMETRY MODEL AND IMPROVED ωkA BASED ON WAVENUMBER COORDINATE ROTATION

The geometrical model of broadband squint SAS imaging is shown in Figure 1. (x,r) is the slanting plane, the x-axis represents azimuth, and the r-axis represents distance. The sonar travels in the positive x-axis at the velocity V_x, the position of the sonar is denoted by X , the beam width is q_{BW} , the beam center oblique angle is $q_{r,c}$, the closest distance between the point target P and the sonar track is R_0, the slant distance of the center of the beam sweeping the target is R_r, suppose that point B is the center point of the selected imaging zone, and the distance between point P and B in azimuth direction is X_n . $R_{\min}$ is the closest distance to the strip region, and $R_{\max}$ is the farthest distance of the strip region. Assuming when the center of the beam sweeps point B, the position of the sonar is the coordinate origin of the synthetic aperture process.

As shown in Fig. 1, the demodulated echo signal is,

$$ss\left(t,X;R_r\right)= A_0 w_r\left(t - 2R(X;R_r)/c\right) w_a\left(X\right)\exp\left\{- j4pf_0R(X;R_r)/c\right\}\exp\{jpm(t - 2R(X;R_r)/c)^2\} \quad (1)$$

Where $w_r(X)$ is the envelope of the transmitted signal, $w_a(X)$ is the azimuthal envelope, f_0 is the carrier frequency, c is the wave velocity, and μ is the transmit signal frequency modulation rate. Using the POSP, the echo signal can be transformed to the range wavenumber domain and the azimuth space, and after ignoring the amplitude term,

$$Ss(K_R,X;R_r) = W_r(K_r)w_a(X)\exp\left\{jK_R R(X;R_r)\right\}\exp(-j\frac{K_r^2c^2}{16\pi\mu}) \tag{2}$$

Where $K_R = \dfrac{4\pi(f_0+f_t)}{c}$, $K_{Rc} = \dfrac{4\pi f_0}{c}$, $K_r = \dfrac{4\pi f_t}{c}$, $K_R = K_{Rc} + K_r$.

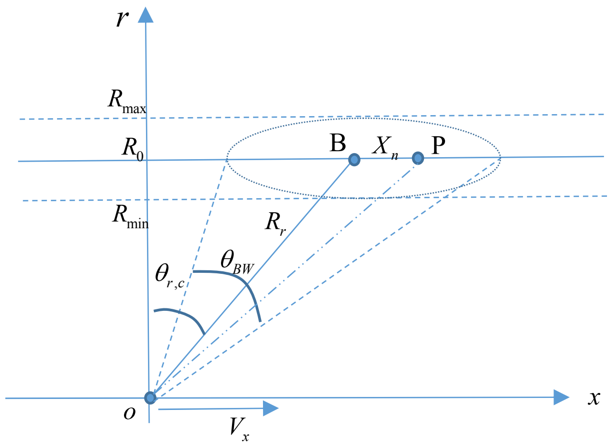


Figure 1 Geometric relationship of squint SAS

In the squint condition, the Doppler center is first zeroed in the range wavenumber domain and azimuthal space domain. According to the Fourier transform nature, the formula (2) multiplies the correction term $\exp(-jK_R\sin\theta_{r,c}X)$ to complete the Doppler center zero processing[6], namely,

$$Ss'(K_R,X;R_r) = W_r(K_r)w_a(X)\exp\left\{jK_R R(X;R_r)\right\}\exp(-j\frac{K_r^2c^2}{16\pi\mu})\exp(-jK_R\sin\theta_{r,c}X) \tag{3}$$

The expression after further transforming the equation (3) to the two-dimensional wave number domain is,

$$SS_0(K_R,K_{x\_base}) = W_r(K_r)W_a(K_{x\_base})\exp(-j\sqrt{K_R^2-(K_{x\_base}+K_R\sin\theta_{r,c})^2}R_0 - j(K_{x\_base}+K_R\sin\theta_{r,c})(X_n+R_r\sin\theta_{r,c}) - j\frac{K_r^2c^2}{16\pi\mu}) \tag{4}$$

From (4), it can be seen that the Doppler centroid processing only changes the support region of the azimuth Doppler wavenumber, and the expression of the two-dimensional wavenumber domain is consistent with that before zeroing.

At this point, the azimuthal wavenumber center is zeroed in the range wavenumber and azimuthal space domain. Analysis of the center of the wavenumber correction term $\exp(-jK_R\sin\theta_{r,c}X)$ shows that the Doppler center frequency zeroing process causes the spatial point target of the range direction to produce a translation with distance $X_t\sin\theta_{r,c}$, where X_t is the azimuth position of the imaging area object relative to the center point object . For example, the distance to the point target located at $(X_n + R_r\sin\theta_{r,c}, R_0)$ becomes $R_0 + X_n\sin\theta_{r,c}$. At the same time, the azimuth position in the two-dimensional wavenumber domain is coupled with the range wavenumber, which can be seen in formula(4).

The method of rotation in the wavenumber domain can effectively solve the problem of position change after Doppler centroid processing and the coupling of range wavenumber and azimuth position. Finally, it can be interpolated to obtain the equivalent side-looking data under the new coordinate system. Coordinate rotation is shown in Figure 2.

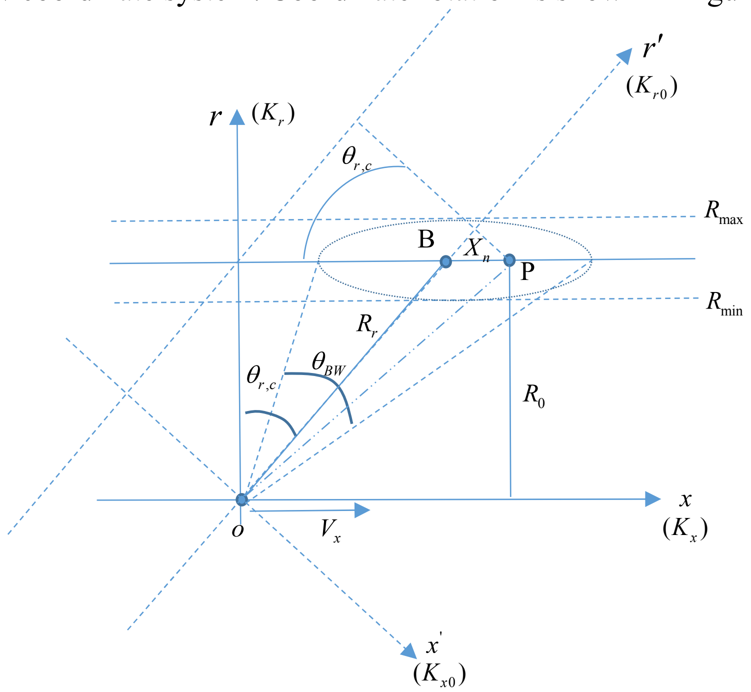


Figure 2 Wavenumber domain coordinate rotation diagram

We construct the two-dimensional space rotation matrix to complete the mapping of the original two-dimensional wavenumber spectrum to the new two-dimensional wavenumber spectrum.

Let $K_r = \sqrt{K_R^{\ 2} - (K_{x\_base} + K_R \sin q_{r,c})^2}$, $K_x = K_{x\_base} + K_R \sin q_{r,c}$, $X_n^{\ '} = X_n + R_r \sin q_{r,c}$, the formula (4) can be expressed as:

$$SS_0(K_r,K_x) = W_r(K_r)W_a(K_x - K_R \sin q_{r,c})\exp(- jK_r R_r \cos q_{r,c} - jK_x X_n^{\ '} - j\frac{K_r^{\ 2}c^2}{16pm}) \tag{5}$$

To construct the rotation matrix of two-dimensional space coordinates, the relationship between (K_{x0},K_{y0}) and (K_x,K_y) is:

$$\begin{bmatrix} K_{x0} \\ K_{r0} \end{bmatrix} = \begin{bmatrix} \cos q_{r,c} & - \sin q_{r,c} \\ \sin q_{r,c} & \cos q_{r,c} \end{bmatrix} \begin{bmatrix} K_x \\ K_r \end{bmatrix} \tag{6}$$

which is,

$$K_{x0} = \cos q_{r,c} K_x - \sin q_{r,c} K_r = \cos q_{r,c}(K_{x\_base} + K_R \sin q_{r,c}) - \sin q_{r,c}\sqrt{K_R^{\ 2} - (K_{x\_base} + K_R \sin q_{r,c})^2} \tag{7}$$

$$K_{r0} = \sin q_{r,c} K_x + \cos q_{r,c} K_r = \sin q_{r,c}(K_{x\_base} + K_R \sin q_{r,c}) + \cos q_{r,c}\sqrt{K_R^{\ 2} - (K_{x\_base} + K_R \sin q_{r,c})^2} \tag{8}$$

From (7) ,(8) can be obtained,

$$K_{r0} = \sqrt{K_R^{\ 2} - K_{x0}^{\ 2}} \tag{9}$$

Therefore, in the new coordinate system (x',r'), the two-dimensional wavenumber field (K_{x0},K_{r0}) of the echo can be expressed as:

$$SS_1(K_{r0},K_{x0}) = W_r(K_{r0})W_a(K_{x0} - K_R \sin q_{r,c})\exp(- jK_{x0}X_n \cos q_{r,c} - j\sqrt{K_R^{\ 2} - K_{x0}^{\ 2}}(R_r + X_n \sin q_{r,c}) - j\frac{K_r^{\ 2}c^2}{16pm}) \tag{10}$$

At this point, the two-dimensional echoes in the wavenumber domain become a side-looking pattern. The new azimuth wave number can be interpolated firstly, and the wavenumber spectrum corresponding to the new azimuth wavenumber can be obtained by the formula (7), that is

$$K_{x\_base} = K_{x0}\cos q_{r,c} + \sin q_{r,c}\sqrt{K_R^{\,2} - K_{x0}^{\,2}} - K_R \sin q_{r,c} \tag{12}$$

And then use the classical STOLT interpolation to calculate the wave number spectrum corresponding to the distance wave number $K_{r0} = \sqrt{K_R^{\,2} - K_{x0}^{\,2}}$. The expression of the echo on the two-dimensional wavenumber domain is,

$$SS_2(K_{y0},K_{x0}) = W_r(K_{r0})W_a(K_{x0} - K_R \sin q_{r,c})\exp(- jK_{x0}X_n \cos q_{r,c} - jK_{y0}(R_r + X_n \sin q_{r,c}) - j\frac{K_r^{\,2}c^2}{16pm}) \tag{13}$$

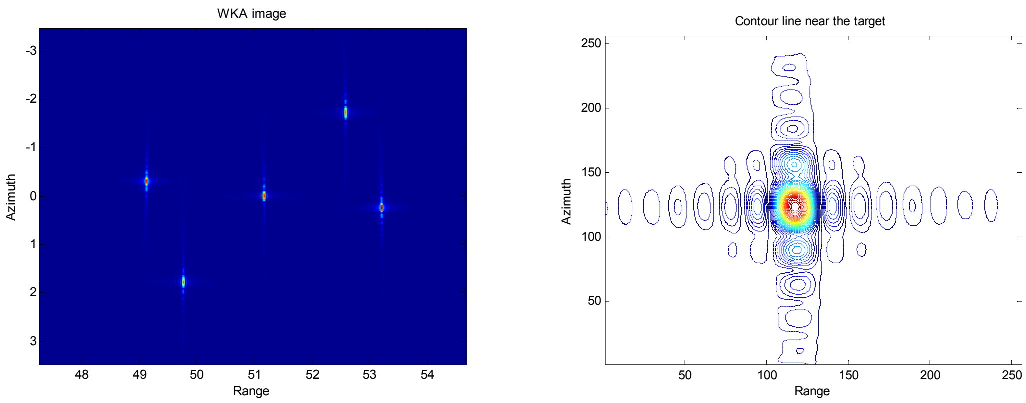
The coupling problem between the range wavenumber and the azimuth space is solved by the rotation of the wavenumber domain. The two-dimensional wavenumber data are transformed into the linear phase term of the coordinate position in the direction of the range wavenumber and the azimuth wavenumber, and the image focus processing can be performed by a two-dimensional IFFT. Further, the spatial coordinate rotation of the image is performed to eliminate the rotation of the target position due to the rotation of the wavenumber domain coordinates, which belongs to the category of image processing and can be realized by simple coordinate rotation in image processing, we will do not go into details.

3. SIMULAITON

We choose a typical parameter of broadband squint SAS, and verify the validity of the improved RMA by processing the point target echo simulation data. The SAS parameters are shown in Table 1.

Center frequency	150kHz	Aperture of element	0.2m
Speed of sound	1500m/s	Platform speed	0.4m/s
Transmit signal bandwidth	20kHz	Squint angle	20°
Pulse Width	20ms	Pulse repetition interval	0.1s

Table 1 SAS system main parameter settings



(a)Target imaging results *(b)Contour lines near the target*

Figure 3 The imaging results of improved algorithm

In Figure 3(a), the coordinate position of object $T_1 \sim T_5$ after imaging is rotated, which is consistent with the algorithm analysis in this paper. The rotation can be eliminated by image correction. Figure 3 (b) shows that the target main lobe and side lobes are distinctly separated from each other and are in the shape of a standard cross, consistent with the contour of the side-looking pattern, indicating the good focusing effect.

4. CONCLUSION

This paper presents an improved ωkA algorithm for broadband squint SAS imaging. Firstly, Doppler centroid correction is performed in the range wavenumber domain and azimuth space. Then, a new wavenumber domain method is constructed by using coordinate rotation to transform the two-dimensional wavenumber domain data into equivalent side-looking data. Finally, two-dimensional inverse Fourier transform completes image focusing. The algorithm avoids the large amount of zeros and data movement needed by the classic ωkA in handling squint data, improves the utilization of echo data, and provides a new idea for SAS imaging processing of strabismus data. At the same time, the Doppler centroid can be corrected by Doppler central mean by the method when the Doppler center is changing rapidly, and the absolute change of the Doppler center can be adjusted to the variance of the mean value,The method reduces the azimuth sampling rate requirement, reduces the number of zeros when the azimuth spectrum expands,which simplifies the imaging process when the Doppler center changes rapidly. When the angle of squint angle is increased, the amplitude of the sidelobe increases due to the influence of the precision of Sinc interpolation, but it can be suppressed by windowing. At present, there are many interpolation algorithms used for STOLT interpolation, but the effect is not very satisfactory, resulting in image results subject to a certain degree of restrictions, which calls for further studies.

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